Emerson (W.) k

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THE ART OF SURVEYING, OR MEASURING LAND.

CONTAINING,

I. The Work to be Perform'd in the Field, as taking all neceſſary Dimenſions and Meaſures.

II. Concerning Protracting, Reducing, Caſting up, and Dividing.

III. Meaſuring inacceſſible Heights, and Diſtances.

Diſce ſed a doctis; indoctos ipſe doceto. CATO.

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THE PREFACE.

THE Art of Surveying is of great antiquity; and it has been ſaid, that it was firſt found out in Egypt. *For when the* Nile, *by its frequent inundations, overflowed the country, and deſtroyed all the boundaries of mens eſtates, or buried them under the ſlime and mud; they were neceſſitated to ſeek out ſome methods or other, whereby to ſet out to every man his due quantity of ground; that it might be reſtored to him without loſs. And as this happened very often, they were under a neceſſity of ſtudying this art, for preſerving the limits and extent of their lands and poſſeſſions.*

But though the knowledge of ſuch an art muſt be neceſſary on this occaſion, and accordingly made uſe of for that purpoſe; yet it might be found out ſooner. For without doubt, the Art of Surveying had been known, ever ſince there had been any ſuch thing as the dividing of lands. For as there could be no ſetting out, or making any diviſion of lands, without taking the meaſures of the whole, and of the ſeveral parts; ſo as ſoon as that happened, muſt the way of meaſuring be known; and thus we meet with a man meaſuring Jeruſalem, Zechariah, ch. 2. *And the ſame neceſſity that cauſed men to divide the land into ſhares, at the ſame time forced them to ſeek for methods to reduce them to an equality, and to find the proportion between one part and another. And thus the firſt dividing*

ding of the land of Egypt *among the people, gave riſe firſt to Surveying and Geometry.*

It has been objected, that the ſurveying of land makes the poor tanants to have dear bargains, and occaſions things to ſell dear and grow ſcarce; and therefore land ought not to be meaſured. But then a man muſt be kept ignorant of his property, in other caſes as well as this. Does a man's knowledge abridge his conſcience? We often ſee that men, by over-eſtimating their eſtates, let their tenants hard pennyworths, being ignorant of the true quantity of ground they let, for the want of ſurveying. But every man, in any ſort of buſineſs, will always make the beſt he can of his goods. And muſt every man have the liberty to make the beſt of his own, and only the owners of land tied from it? Now the Art of Surveying teaches how to give every man his due, and to do juſtice to every body, to the landlord as well as the tenant, to the buyer as well as the ſeller; by ſhewing the true meaſure and quantity of the land. Therefore Surveying is a neceſſary and uſeful Art, which not only gives the true quantity, but ſhews rules for laying down maps and drafts of an eſtate; which exhibits to view every field, and every object therein, in their proper ſituations; which is a uſeful invention.

Many large volumes have been writ concerning the Art of Surveying, by which it has been drawn out to an unneceſſary length. For the whole of it is founded upon the ſimpleſt principles of Geometry, which are eaſily learned, and cannot need ſuch long and tedious explications; for ſuch methods only confound the reader. For no ſubject ought to be treated more voluminouſly, than there is real difficulty in it; which is not to be found in this Art. It is a good old maxim, μέγα Βίϐλιὸν μέγα κακὸν, *and ought always to be obſerved. For by ſpinning out things in long diſcourſes, the reader is loſt in a miſt of words. But this perplexity is avoid-*

ed, by treating the ſubject in a conciſe way, and going no farther than the ſubject leads.

In the following Book, you have a verbal deſcription of the inſtruments uſed in Surveying. For it is not worth the while to give any Cuts of them; a ſight of them being more inſtructive than any figure can be. In Sect. I. *you have an account of all the work that is to be done out of doors; with proper Rules to proceed by.* Sect. II. *teaches how to plot, reduce, caſt up, and divide any parcel of land. And laſtly,* Sect. III. *ſhews the methods of taking heights and diſtances, both acceſſible and inacceſſible; which is a curious and uſeful ſubject, and very neceſſary to be known.*

I have omitted nothing belonging to this Art, and have treated of every thing in as conciſe a manner, and yet as clearly, as the ſubject would admit of. And ſo I take my leave of the reader, wiſhing he may meet with all the ſatisfaction he expects.

W. Emerſon.

SURVEYING.

SURVEYING, or *meaſuring Land*, is the art of finding the content of any field, or parcel of land; which is always expreſſed in acres and decimal parts; or perhaps in acres, roods and perches. This ſort of meaſuring differs from others, as it is concerned in nothing but meaſuring parcels of the earth's ſurface, which are large figures; and whoſe meaſures are taken in chains and decimal parts; whereas the dimenſions in other ways of meaſuring are taken, and caſt up, in inches, feet, yards, &c.

The inſtruments uſed in ſurveying are of ſeveral ſorts. 1. Thoſe for meaſuring angles; as the *Theodolite*, *plain table*, *ſemicircle*, *circumferentor*, *Geodetical ſtaff*, *infallible*, &c. 2. For meaſuring diſtances; as the *wheel*, or *perambulator*, the *chain*, &c. 3. For levelling; as the *water level*, *ſpirit level*, &c. 4. For making draughts upon paper; as *ſcales*, *line of chords*, *protractor*, *reducing ſcale*, *compaſſes*, *parallel ruler*, &c. 5. For marks, &c. as *arrows*, *ſtation ſtaves*, *ſtreamers*, &c.

The Theodolite.

This is a braſs circle, 12 or 14 inches diameter; its circumference is divided into 360 degrees; it has two ſights upon it, fixt at the beginning of the degrees; and alſo a moveable index, with two ſights, which moves about the center. The whole is to be fixt upon a three legged ſtaff, with a ball and ſocket. To ſome of theſe inſtruments are fitted teleſcopic ſights; which makes it one of the

beſt

beſt inſtruments for taking angles. And ſome of them have a card and compaſs, with a needle playing round the center, which in ſome caſes is very uſeful.

The plain Table.

This is a flat board or table, the bigneſs of a ſheet of paper; fitted into a frame going round its edges, which frame is to keep the ſheet of paper faſt on the table, for uſing in the field. The edge is graduated into degrees, from a braſs center in the middle of the table; by means of which it may be uſed without a paper. It has a looſe index with two ſights, being ſighted both back and forward; and upon this index is a tangent line for taking heights. The index is made to turn upon any given point, againſt a pin, or the point of a pair of compaſſes placed therein, as a center; and in taking angles, this center muſt be the braſs center. The ſtreight edge of the index is called the *fiducial edge*.

The uſe of this inſtrument is to draw the plot or figure of any field or parcel of land upon a paper, in the field. For when the work in the field is finiſhed, you have the exact form of the ground upon the paper on the plain table. This is a proper inſtrument for ſurveying ſmall incloſures, and of great diſpatch; but not ſo good for large eſtates, by reaſon the paper upon the table will contract in dry weather, and expand in moiſt weather. And therefore if one part be drawn when it is dry, and another when it is moiſt, the field will be diſtorted, and the parts out of proportion.

The under ſide of the frame is divided into equal parts, and may be turned upſide down upon occaſion.

Circum-

Circumferentor.

This is a parallelogram, eight inches long and four broad, made of box or braſs, its edge graduated or divided into degrees; to this belongs an index with ſights forward and backward. It may alſo be made in form of a circle eight inches diameter.

To this belongs a box and needle; and therefore it is the fitteſt inſtrument for mapping a harbour, and for drawing the poſition of rocks, ſands, ſhoals, ſoundings, &c. And proper for meaſuring a county, if it be made large.

Semicircle.

This is a large ſemicircle divided into degrees, having ſights placed at the beginning or edge. It has a moveable index with ſights back and forward, moving about the center. To this is fixed a box and needle; it is the ſame as half a Theodolite. It is fitted to a ball and ſocket, and ſet upon a three leg'd ſtaff, like all the reſt.

Geodetical Staff.

This is an inſtrument with two legs, having ſights upon them, opening upon a braſs center pin, like the legs of a pair of compaſſes; each leg is a foot and half long, or more. There is a long ruler, called the *Graduator*, which turns upon a pin in the end of one of the legs, and goes thro' the end of the other leg. This ruler is graduated with a line of chords, anſwerable to the length of the legs, which is the radius. As the inſtrument is opened, the moveable leg, running along the graduator, ſhews to what angle the inſtrument is opened. And when it is open, it appears in form of an iſoſceles triangle; and may be opened to a right angle, or ſomething more; and then the graduator will be the hypothenuſe. This is an excellent inſtrument for taking angles.

Infallible.

This is an inſtrument not very common; it is a flat board, or a plate of copper or braſs, a foot ſquare. On the middle or thereabouts is fixed a perpendicular pin, on which is ſcrew'd an index with two ſights. The table or plate is to be covered with a paper, which muſt be folded at the ends under the inſtrument, and ſcrew'd faſt at every corner, by four plates fixt underneath the table. Then the index muſt be put upon the pin, which will come thro' the paper.

This inſtrument is very ſimple, and very eaſily made, and differs nothing from the plain table, but only that it is ungraduated; and the index always turns upon the fixt pin. When it is uſed in the field, it muſt be placed on a three legged ſtaff, being a ſtaff with three legs opening outward, to ſtand on the ground in any poſition you will. All theſe inſtruments are for taking angles or draughts.

Wheel, or Perambulator.

The wheel to meaſure diſtances, is 2$\frac{3}{4}$ yards in circumference, but ought rather to be 5$\frac{1}{2}$ yards about, or a ſtatute pole. It conſiſts of clockwork, and on the face has two hands or pointers, to ſhew the diſtance run. The wheel runs between two cheeks, in a frame of wood; by taking hold of the frame it is driven forward, and by the motions of the hands, and diviſions on the face, is ſhewn how many yards, poles, furlongs or miles, you have gone. One hand goes round in half a mile, and the other in 12 miles.

This inſtrument is very uſeful for meaſuring roads, rivers, and all level ground.

Chain.

There are ſeveral ſorts of chains, but *Gunter's Chain* is the beſt for meaſuring land; its length is 4 poles or 22 yards; and is divided into 100 links; and at every 10 links there is a piece of braſs, with figures or tails for diſtinction; by theſe, the links are eaſily reckoned.

In meaſuring towns, a chain of 50 feet and 50 links is the moſt commodious.

Water Level.

This is a round tube made of wood or braſs, about three feet long; the ends of it are turned up ſquare; in which are cloſe cemented two glaſs tubes perpendicularly. The tube is to be almoſt quite filled with water. There are two ſights fixt to this machine; and often teleſcopic ſights, for great diſtances. When it is fixt level on a three legged ſtaff, the water will riſe to equal heights in both the glaſs tubes, where proper marks or diviſions are made to ſhew it.

The uſe of theſe is to find a true level, for conveying water, draining fens, &c.

Spirit Level.

A ſpirit level, or air level, is a cylindrical glaſs tube, very near filled with ſpirit of wine, wanting only a drop, and then ſealed. On this, ſights are fixt, which ſhould be teleſcopic ſights. When the ſights are placed level or horizontal, the bubble or drop of air will be in the middle; but when it is not level, the bubble will go to the higher end.

In the beſt ſort, this cylindric glaſs tube, with the bubble, is fixt upon the tube of an aſtronomical teleſcope, two or three feet long; and ſo adjuſted,

juſted, that when the bubble is in the middle, the object or mark, ſeen through the teleſcope, is juſt level with it. And there is a proper apparatus with ſcrews to ſet the whole level; which is known by the bubble being in the middle.

Scales.

The ſcales or rulers ought to have ſeveral lines of equal parts upon them; ſome greater, ſome leſſer, to ſuit all caſes, particularly diagonal ſcales, which ſhew the diviſion of 1 into a hundred parts. Thus, we muſt have a ſcale of a chain in an inch, a chain in $\frac{3}{4}$ of an inch, in $\frac{1}{2}$ an inch, &c. By theſe we draw a field, or a *map* or *plan*, called alſo a *plot* or *plat*.

There are alſo ſcales made that one may prick off the diſtances by the edge.

Line of Chords, Protractor, reducing Scale, Compaſſes, parallel Ruler.

Theſe are ſo well known, that they need no deſcription. The chords, and protractor, are for laying down angles; the reducing ſcale, for increaſing or diminiſhing a plan or map; the compaſſes, for ſetting off diſtances; and the parallel ruler, for drawing parallel lines. As to the reducing ſcale, it turns on a center pin, nearer one end than the other. On this are ſeveral lines of equal parts, running both ways from the center; and in certain given proportions.

There is alſo an inſtrument made with two drawing pens, ſo that tracing one of them over the old draught, the other will, on a clean paper, deſcribe it anew, in a leſs or bigger form; being a very expeditious inſtrument.

Arrows.

Theſe are ten ſmall ſticks, about a foot or half a yard long, and ſharp at one end, to ſtick in the ground at the end of every chain, in meaſuring. They may be tipt or piled with iron at bottom; and coloured blue and red near the top, to make them viſible.

Station Staves, and Streamers.

Theſe are long poles to be ſet up in the angles of a field, or in other remarkable places. If they be far off, they have moveable vanes or ſtreamers placed on them, to be ſlid up and down; or tied to their tops. And theſe vanes or marks may be white, or coloured red or blue, to make them viſible at great diſtances.

Some of theſe poles are divided into links, for meaſuring offsets readily.

The Field Book.

As it is not poſſible to ſurvey any great quantity of ground without noting down the particulars; therefore you muſt have in the field a little book, called the *Field Book*, in which to write your ſtations, offsets, angles, lines, obſervations, &c. This book you may divide into columns, as you think convenient, as will beſt ſuit your occaſions: for there are hardly two perſons that have the ſame method for their field notes; however, it ought to be contrived in the eaſieſt and moſt intelligible manner; for any method, that lays a burthen upon the memory, is imperfect, and not fit to be practiſed.

This book may be contrived after the following manner. Make three columns, the middle one for the angles (mark'd <), ſtations (mark'd ⊙), the diſtances meaſured, and bearings, &c. The two columns on the left and right, are for the offsets on the left and right, againſt their reſpective diſtances in the ſecond column; and for ſuch remarks as occur. Draw a line under the work at the end of every ſtation; that the workings at the ſeveral ſtations may not confound one another.

Note, in ſurveying with the plain table, you need no field book; the whole work being deſcribed upon it, except offsets, which may be ſet thereon, as they happen.

The

The FIELD BOOK.

Offsets and remarks on the left.	Stations.	Offsets and remarks on the right.
	⊙ 1	
.12	< 113° 15	
	00	.26 Thomſon's hedge begins.
	56	touch the hedge
	4.13	.34
.32 a croſs hedge	6.09	.41 a tree
	8.00	.00
	⊙ 2	
	< 87 43	
	.00	.00
	1.80	Gutter changes
.87 houſe corner	3.15	
	4.82	.27 a pond
	⊙ 3	
	64 15	
a gate	0 . 84	.5
	2 . 75	.15
	4 . 11	.29 a gate
.88 a brook	5 . 28	
18 foot path	5 . 92	.15
	6 . 00	00
15 hedge corner	7 . 54	
	9 . 20	28 a ſpring.

Explanation.

⊙ 1 is the firſt ſtation, and the < there is 113 15. on the right an offset .26 or 26 links, where Thomſon's hedge begins; meaſuring to .56 we touch the hedge. At 4.13 an offset of 34. At 6.09 an offset .41 at a tree. At 8.00 no offset. On the left, at firſt an off-ſet .12; at 6.09 an offset .32 where there is a croſs hedge. And ſo for the reſt of the ſtations.

1. A

1. *A Table of Long Meaſures.*

7.92 inch	1 link						
12	$1\frac{17}{33}$	1 foot					
36	$4\frac{6}{11}$	3	1 yard				
198	25	$16\frac{1}{2}$	$5\frac{1}{2}$	1 pole			
792	100	66	22	4	1 chain		
7920	1000	660	220	40	10	1 furl.	
63360	8000	5280	1760	320	80	8	1 mile

2. *A Table of Square Meaſures.*

9 feet	1 yard					
$272\frac{1}{4}$	$30\frac{1}{4}$	1 perch				
4356	484	16	1 chain			
10890	1210	40	$2\frac{1}{2}$	1 rood		
43560	4840	160	10	4	1 acre	
27878400	3097600	102400	6400	2560	640	1 mile

In theſe Tables, each perpendicular column is of one denomination throughout. And all the lateral ones are equal, but of different denominations.

Note, a pole and perch is the ſame thing, and a ſquare chain is called a rape in the North, being the tenth part of an acre.

SECT. I.

The Practice of SURVEYING, *containing the Work to be done in the Field.*

PROB. I.

To take an angle BAC, *or meaſure the quantity of it in degrees.* 1.

SET up two ſtation ſtaves perpendicular, in ſome parts of the lines AB, AC, as at B and C; and put thereon ſome viſible marks, large enough to be ſeen from A. Then plant the inſtrument in the angular point A, or as near it as poſſible. Then,

1. For an inſtrument with a paper upon it, as the plane table, and infallible. Screw the table faſt; then direct the index to one mark as B, and turning it about, till thro' the ſights you ſee B, then draw the line AB by the edge of the index. Then direct the index to the other mark C, and turning it about till you ſee it thro' the ſights, then draw the line AC by the edge of the index. Then you have the angle BAC upon the paper. Here the edge of the index muſt be kept all the while upon the point A.

2. For inſtruments with two fixt ſights, and a moveable index, as the theodelite, ſemicircle, geodetical ſtaff, &c. direct the fixt ſights along one of the lines AB, turning the inſtrument about, till thro' the ſights you ſee the mark B, there ſcrew faſt the inſtrument. Then direct the moveable index along the other line AC, and move it about,

Fig. 1. till thro' the ſights you ſee the mark C. Then the degrees cut by the index, upon the graduated limb of the inſtrument, ſhews the quantity of the angle; which ſet down in the field book. If the angle be obtuſe, you muſt take it by back ſights; cauſing *b*, *c* to lye on the other ſide of A.

3. By an inſtrument with a needle. Place the inſtrument ſo, that the north end of the needle may hang directly over the flower-de-luce. Then direct the ſights to one mark as B, and note the degrees cut by the needle. Then direct the ſights to the other mark C, and note again what degrees are cut by the needle. For their ſum or difference (as the caſe is) will be the quantity of the angle BAC; which ſet down.

If either mark cannot be ſeen from A, as in the caſe of hills, trees, &c. ſet up a new mark in ſome place between, from whence both A and the other may be ſeen. And to place it in the line, ſhift it to the right and left, till both A and the other mark appear through the ſights, both forward and backward; and that is the place of the new mark, which may be uſed inſtead of the other.

Note, if ſeveral angles are to be taken at one point as A; take them all from one and the ſame line as AB. For by this means, there will be leſs danger of committing any error.

SCHOL.

There is alſo a way to take an angle in the field by the chain only; which is this. Take a chain length along both ſides AB and AC, as A*b* and A*c*; then meaſure the diſtance *bc* between *b* and *c*, and write it down. Then the angle A may be found by plain trigonometry; for it will be as A*b* : radius : : ſo $\frac{1}{2}$ *bc* : S. $\frac{1}{2}$ the angle A. Or find $\frac{1}{2}$*bc* in the table of natural ſines, and it ſhews half the angle BAC.

But this method being ſubject to errors, ought Fig.
not to be practiſed, but in caſe of neceſſity, or for want of proper inſtruments.

PROB. II.

To meaſure a right line, or any diſtance, upon the ground.

1. Let AB be the diſtance to be meaſured. Set 2.
up ſtaves with marks upon them, at A, B and C. Then the leader of the chain muſt take the ten arrows in his left hand, and take hold of the end of the chain with his right hand, by putting his hand into the ring. Then let him go from A towards B, drawing the chain after him; and as he goes, let him take an arrow into his right hand ready to ſtrike down. The follower muſt ſtand at A, having hold of the other end of the chain in his right hand; and cauſe the leader to go always along the right line AB, till he comes at the ſtretch of the chain; then the follower muſt hold his end cloſe to A, and the leader, ſtretching the chain ſtreight, muſt prick the arrow upright into the ground, at the very end of the chain. Then he muſt go forward, and take another arrow into his right hand; and when the follower comes to the arrow ſticking in the ground, he muſt ſet the end of the chain cloſe to it, and take it up with his right hand; whilſt the leader, at the ſame time, pulls the chain ſtreight, and pricks in another arrow; and then proceed as before; and whilſt the follower is going, he muſt put the arrow into his left hand. Thus at the end of every chain, the leader puts down an arrow, and the follower takes one up. And thus they go on, till they come at B, the end of the line; and then the follower muſt reckon his arrows, for there will be ſo many chains;

Fig. 2. and if any thing to ſpare, ſee how many odd links reaches from the laſt arrow to B; and ſet all down, the chains in whole numbers, and the links in decimals.

If all the arrows are gone before the leader gets to the end at B, he muſt come back for the ten arrows; and the follower muſt ſet his toe at the place where the laſt ſtick or arrow was; till the leader, pulling the chain ſtreight, puts in another ſtick; and then proceed. In great diſtances, you will have the arrows to change ſeveral times; then you muſt note the number of changes, which will be eaſieſt done thus. Put 20 ſmall ſtones in your pocket, and at every change of the arrows (that is, at every 10 chains) remove a ſtone into another pocket. And at laſt, counting the ſtones removed, you will have the number of changes; where 8 changes make a mile. And then the whole length muſt be ſet down in the field book.

In going from A to B, the leader muſt always keep his eye on the mark at B, and keep in the line AB, ſo as to cover the mark at B from the follower.

At the end of the line AB, and at the end of every change, count all the arrows in both parcels, to ſee if they both make ten; if they do not, one has been dropt, and the diſtance muſt be meaſured over again.

2. In meaſuring to or from any hedge, you muſt account 5 links, from the roots of the quicks, on that ſide where the gutter is; and that will be the boundary; but in walls, the outſide is the boundary.

3. When the mark at B cannot be ſeen from A, then ſet up a mark in ſome place between, from which both A and B may be ſeen; and this place may be found, by the inſtrument, placing it there, and removing it, till A and B both appear through

the ſights backward and forward. And if one will not do, ſet up two or three marks after the ſame manner. Fig. 2.

4. If the diſtance be very great, as amounting to ſeveral miles, then the eaſieſt way is to meaſure it with the wheel; in which there are ſeveral indexes, ſhewing how many miles, furlongs, &c. have been paſſed over. But this is not ſo exact as the chain.

5. Great diſtances alſo may be meaſured expeditiouſly, by the firing of great guns in a dark night; and obſerving with a watch or a pundulum vibrating ſeconds, how many ſeconds there are between the flaſh, and the report of the gun. For that number of ſeconds multiplied into 1142, gives the number of feet in the diſtance. But this is not ſo exact as the former methods.

6. If the line you meaſure is not horizontal, but riſes or falls; and you want the horizontal length. Then with a quadrant, you muſt take the elevation or depreſſion of that line; and then the horizontal length may be found by trigonometry.

Or in deſcents, or going down a ſteep place, you may find the horizontal level thus. At the top, ſtretch out as much of the chain as you can horizontally, and let a ſtone fall from that point of the chain, and where it falls, ſet that point of the chain; and ſtretch out another part of it horizontally, from which let fall another ſtone, which will ſhew another place to hold at again horizontally; and proceed thus to the bottom. Then the ſum of all theſe parts of the chain is the horizontal line.

PROB. III.

To meaſure offsets, and find their ſituation upon any right line.

3. Let AB be any right line, and A*oegh* a crooked hedge; *co*, *de*, *fg*, B*h*, &c. ſeveral offsets, to be meaſured. From every angle or ſenſible bend of the hedge, as *o*, *e*, *g*, &c. let perpendiculars be ſuppoſed to fall on the line AB, as at *c*, *d*, *f*, &c. which points may be taken near enough, by the eye, if the offsets are not very large; but if they are, it muſt be done by the inſtrument. To this end, you muſt have a croſs line upon your index, perpendicular to the viſual ray. Then as you meaſure along the line AB, when you come at *c*, ſet down the length A*c*, and meaſure the offset *co*; and to know when you are at the point *c*, place your inſtrument about *c*, and look thro' the ſights to A and B, and if you can ſee both A and B, looking back and forward, then you are in the line; if not, the inſtrument muſt be removed till you can ſee both. The inſtrument ſtanding thus, look along the croſs line on the index, and if it run directly to *o*, then *c* is the point where the offſet falls; but if it point not to *o*, the inſtrument muſt be removed back or forward in the line AB, till it does; whilſt the ſights ſtill point to A and B.

Proceeding forward, find the points *d* and *f*, &c. in the ſame manner, where the offsets *de*, *fg*, &c. fall; and meaſure the offsets as you go along. And care muſt be taken to go in the line AB; and that at each point *d* or *f*, the ſights may point to A and B, at the ſame time that the croſs line points to *e* or *g*. Then all muſt be ſet down in the field book, as you meaſured them; that is, both the diſtances along the ſtationary line, and the correſponding offsets, which will be as follows. Fig. 3.

	right hand.
⊙ A	o
$Ac = 1.94$	$.87 = co$
$Ad = 3.42$	$.23 = de$
$Af = 4.17$	$.81 = fg$
$AB = 4.45$	$.47 = Bb$

Cor. *After the ſame manner, if* Aoegb *be an irregular curve; erect* 3, 4 *or* 5 (*equidiſtant*) *ordinates or offſets, at c, d, f,* &c. *and meaſure the baſe* AB, *and the offsets as before.*

PROB. IV.

To draw a ſtreight line on the ground, from one place A, *to another* B.

1. Take a long cord, and faſten one end at A, and carry the other end towards B; if it will reach thither, ſtrain it tight, and faſten it at B. Then either pin down the cord, to the ground; or elſe, which is better, make a trench along the line with a ſharp ſpade, and that will be your line. 4.

2. If the cord will not reach to B; let ſome body draw the cord ſtreight towards B, which ſuppoſe will reach to C; and there let him hold a ſtake upright; and moving it to and fro, ſet it at laſt, that A, C and B may hide one another, when ſeen from A or B; then A, C and B will be in a right line. And after this manner ſet one length after another, till you come at B.

3. If there happen to be any hill or riſing ground in the way, that one place cannot be ſeen from the other; then perhaps, a very long pole may be ſet up in one of the places, and be ſeen at the other; and then proceed as before. But if this cannot be 5.

Fig. 5. done, ſet up two or more ſtakes in the way, as C, D, &c. and let them be ſo placed by trials, that C ſeen from A, ſhall cover D; and D ſeen from C, ſhall cover B or the next ſtake, and ſo on to the laſt. *And note*, in directing one another, no words muſt be uſed, but only ſigns of the hand; for it is ſuppoſed to be too far off, to hear any directions.

But if you have an inſtrument with back ſights as well as fore ſights, fix your inſtrument ſo between A and B, that you may ſee both of them, through back ſight and fore ſight, and there you muſt place a ſtake as at C; but if A and B cannot both be ſeen from C, ſet it by gueſs, and another D alſo by gueſs, and ſo on. Then try them, and ſhift them, till A and D appears from C; and C and B from D, &c. by back ſight and fore ſight; and then they are all in a line.

PROB. V.

To raiſe a perpendicular upon a line AB, *at a given point* A, *upon the ground.*

6. Take a ſquare, or rather a graduated inſtrument with ſights, and placing it at A, direct the ſights of the inſtrument, or the ſide of the ſquare along AB, and fix the inſtrument. Then turn the ſights to 90 degrees, and look thro' them along the line AD, and let ſome perſon ſet a ſtake forward, as at D. Then DA is perpendicular to AB. If you uſe a ſquare, you muſt look along the other ſide of the ſquare, to find a point, as D.

Or thus.

Set off 40 links from A to B, and 50 links from B to D, ſo that 30 links may juſt reach from D to A. Then DA is perpendicular to AB.

For then the triangle BAD is right angled at A. Fig. For the ſquare of the hypothenuſe BD is equal to 6. the ſum of the ſquares of the two ſides, BA, AD.

PROB. VI.

To let fall a perpendicular from a given point D, *upon any line* AB, *upon the ground.*

Set an inſtrument ſomewhere about A, in the line 6. AB; and a mark at B; then directing the ſights along AB, fix the inſtrument. Then turn the index to 90 degrees, and if you ſee D thro' the ſights, you are in the perpendicular. If not, move the inſtrument back and forward in the line AB, till you can ſee D, with the index ſet to 90 degrees; the inſtrument having the poſition before deſcribed; that is, directed to B; then DA is a perpendicular. This may be done in a few trials.

A ſquare will perform the ſame thing; moving one ſide along AB, till the other ſide point to D, as at A.

PROB. VII.

To lay down a given angle on the ground, as CAB.

Set up a ſtake ſomewhere in the line AB, as at B; 7. then place an inſtrument at A, and direct the ſights at the beginning of the degrees, to B; then fixing the inſtrument, ſet the index to the degrees of the given angle; as DAE, and looking along the index AD, cauſe ſome aſſiſtant to ſet up a ſtake in the line ADC, as at C; then the angle CAB will be that required.

By an inſtrument with two legs; ſet it to the given angle, and placing it at A, direct one leg AE along the line AB, to the ſtake at B; and ſet up another

Fig. 7. another ſtake in the line AC, as at C, in the direction of the other leg AD.

PROB. VIII.

Thro' a given point C, *to drawn a line parallel to a given line* AB, *on the ground.*

8. Meaſure the neareſt diſtance of the given point C from the line AB; and at a convenient diſtance from A, as at B, raiſe the perpendicular BD to the line AB. Set the length AC from B along the perpendicular to D, and draw CD, which will be parallel to AB.

Or thus.

Take any ſtation B in the line AB; and with a proper inſtrument, meaſure the angle ABC. Then, by the laſt Prob. lay down the angle BCD equal to ABC; draw CD, which will be parallel to AB.

Or at any point A in the line AB, take the angle CAB; then going to C, ſet off the ſame angle ECD, to D; E being in the line CA.

Otherwiſe thus.

Set up a ſtake at C, and another at any place as A, in the line AB. And let two obſervers hang up two lines and plummets at D and B. Then let the obſerver at B take notice when the ſun, or ſome ſtar (agreed on between them) comes in the line BA; and at that inſtant give notice to his correſpondent at D, to ſet his line and plummet ſo, that D and C may be in a line with the ſun or the ſame ſtar. And then CD is parallel to AB. It will be beſt for the obſerver at D, to keep his line and plummet always in a line with C, and the ſun or ſtar.

PROB. IX.

To meaſure the horizontal baſe of a Hill, as BC.

Let BAD be a horizontal line, CA a perpendicular from the top, BC the ſlope or ſide of the hill. Set proper marks up, and with a proper inſtrument, take the elevation, that is, the angle ABC or ACB; and one being given the other is known. Then meaſure with a chain the ſide CB. And it will be, as rad : BC : : S.angle BCA : to BA. 9.

If the hill has different declivities in ſeveral places, take the angles in theſe ſeveral places, and the ſide of each part, as before. Then find the baſe of each, by the ſame operation; and the ſum of all, is the whole baſe.

All this may be done by rule and compaſs, by laying down the triangle BAC, or the ſeveral parts thereof.

If there be likewiſe a deſcent on the other ſide, as CD; you may find the part AD in the ſame manner as before. Or proceed thus, take the angles at B and D, and ſubtract their ſum from 180 gives the angle BCD; or the angle BCD itſelf may be meaſured; and meaſuring either ſide of the hill as BC, then, as S.angle D : BC : : S.angle BCD : BD the whole baſe.

Cor. *By the ſame method the height of the hill* CA *may be found. For it will be, as radius* : BC : : S.*angle* ABC : *height* CA.

PROB.

Fig.

PROB. X.

To reduce a field to fewer ſides in the field, where an angle is very obtuſe, or when one of the ſides is very ſhort.

10. Let DEFKCBAD be any field, which here conſiſts of ſeven ſides. Then,

1. Suppoſe ABC is a very obtuſe angle or bending, which we want to take way. Produce the longer ſide of the angle CB to cut the ſide AD (adjoining to the other ſide), at G; then take the point I, ſo that it may divide the line GA, in the ſame proportion, as B divides GC; that is, ſo that GI may be to GA, as GB to GC. And as GA is in this caſe very ſhort, this may be done by the eye exact enough, for the moſt part; or as exactly as the two points A and B can be determined. Then draw the line CI, and leaving out the angles B and A, the two lines CB, BA are reduced to one CH, and the field conſiſts of one ſide leſs than before. And the ſame may be done with the angle FED, by producing FE till it cut AD, and finding the point Q as I was found before.

2. Suppoſe ED a very ſhort ſide. Stand at D the end of the ſhort ſide, and obſerve, directly over the angle F (the next but one), ſome very remote object, as a houſe, tree, mountain, ſun, moon, ſtar, cloud, &c. Then immediately remove along AD, till you ſee the ſame object directly over E; at that place Q, put in a ſtake or ſome mark. Then leave out the angles E, D, and take Q inſtead thereof. And the two ſides FE and ED are reduced to one FQ, which takes away another ſide of the field. And the ſame thing may be done with the ſide BA, by obſerving the ſame remote object from A over C, and from I over B.

The reaſon of this proceſs is this; the lines AC Fig. 10. and IB are parallel; and the lines DF and QE are parallel; and therefore the triangle CBA is equal to CIA; and the triangle FED is equal to FQD.

Cor. *This method is very uſeful, when we meet with a field with crooked hedges, by reducing it to fewer ſides.*

For by this means we have the ſame quantity of ground, and fewer hedges. For the lines CI and FQ take in as much as they leave out.

SCHOL.

When any ſide of a field is very irregular, you may draw a ſtrait line inſtead of it, which ſhall take in as much ground in ſome places, as it leaves out in others; and ſo does not alter the quantity of it. And this way (of giving and taking) ſhortens the work; and ſuch a line may be drawn as exactly by the eye, in moſt caſes, as the boundaries themſelves can be aſcertained.

PROB. XI.

To ſurvey a triangular field ABC.

1. *By the Chain.*

Set up ſtation ſtaves at the angles A, B and C. 11. Then meaſure with the chain from A to C; and when you come at P, where the perpendicular BP falls, ſet up a ſtake there. And the point P is eaſily found, for as you go along from A, ſet your inſtrument where you think the perpendicular will be, then look thro' the ſights back and forward, to ſee that you be in the line AC; that done, look thro' the croſs line upon your index, and if you ſee the vertex B, then you are at the place P, where the perpendicular falls. But if B does not appear, you

Fig. you muſt move back or forward in the line AB,
11. till B can be ſeen along the croſs line, as at P. Then having meaſured AC, and ſet it down; go to P and meaſure the perpendicular PB, and ſet it down likewiſe. And thus you have the baſe and perpendicular, which is ſufficient for finding the content.

Or meaſure all the three ſides AC, CB, BA, ſeverally, with the chain; and note them down. Then the triangle may be laid down, and the content caſt up by Sect. II.

2 *By taking one or more angles.*

Meaſure two ſides AC and CB, and the angle included ACB; by which the triangle may be deſcribed.

Or meaſure one ſide AC, and two angles A and C, then the third angle B will be known.

Or, if you cannot get the angle C; take any point D in the baſe AC, where take the angle ADB, and meaſure AC and DB; and then it may be caſt up by Sect. II.

PROB. XII.

To ſurvey a field in form of a trapezium.

1. *By the chain.*

12. Set up ſtation ſtaves with marks or ſtreamers, at all the corners A, B, C, and D. Then chooſe one of the diagonals AC or DB to meaſure, let it be DB; it ought to be ſuch a one, as that you can ſee the mark at B from D, or the mark at D from B; and likewiſe the marks at A and C from the diagonal DB; and likewiſe the perpendiculars AF and CG ſhould fall within the figure.

Begin at D, and meaſure along the diagonal DB in a right line, till you come at the place of the perpen-

Fig. 12. perpendicular AF, which will be known, by looking backward and forward thro' the ſights to D and B; and then looking along the croſs line to ſee A; removing the inſtrument, till A can be ſeen. Then at F put in a ſtake. Then meaſure forward, and find the place G of the other perpendicular GC as before; and put in another ſtake at G. Then meaſure to the end B, and ſet down the length DB. Then go to F or to A, and meaſure the perpendicular FA, and ſet it down. Laſtly, go to G, and meaſure the perpendicular GC, and ſet it down, with the reſt. And by theſe the content will be found.

Diagonal DB — 8.37
1 perp. AF — 5.18
2 perp. GC — 4.86

Otherwiſe, by the Chain.

Draw a rude draught of the field to imitate it, upon paper; and having ſet up marks at all the corners, draw a diagonal as DB upon your draught. Then begin at any angle D, and meaſure all the ſides round about to D again; and then the diagonal DB; and ſet them all down as they are meaſured, upon the correſpondent lines of your draught. Then from theſe meaſures, the field may be laid down upon paper; and the content found, by Sect. II.

In meaſuring this way, it is beſt to take the ſhorteſt diagonal.

Or thus, by the Chain.

13. Marks being ſet up at A, B, C and D. Find the interſection E of the diagonals AC, DB; thus, place the ſtaff and index at E, and if you can ſee D and B forward and backward, and alſo A and C forward and backward, thro' the ſights, then E is the interſection ſought. But if not, remove the ſtaff,

Fig. ſtaff, till you can ſee them all; which is ſoon done
13. by a few trials. Then meaſure the lines in this order, AE, EC, CB, BE, ED, and DA (for proof), with the chain. And write their lengths on their proper lines, in a rude draught of the field. From theſe meaſures, the field may be laid down, and the content found, by Sect. II.

In all theſe methods, there is no occaſion for any inſtrument (beſides the chain), but an index with two ſights and a croſs line, and a ſtaff to ſet it on; whoſe uſe is to make you go ſtrait, by trying foreſight and back ſight; and for finding the perpendiculars, when they are required.

2. *By taking an Angle.*

Having ſet up marks at all the corners; find the interſection E, of the diagonals in the field, as in the laſt method. Then fixing the inſtrument at E, take any one of the angles at E, as AEB, and ſet it down. Then meaſure the diagonals AC and BD, and ſet them down. Then by theſe, the content will be found, by Sect. II.

Angle AEB,	66°	20′
AC,	7.	54
DB,	6.	17

Or thus, by taking ſeveral Angles.

Make a rude draught of the field upon paper, and draw one diagonal as DB, which meaſure and ſet down in the draught. Then with the inſtrument, take the quantity of the angles, DCB, CDB, and ADB, DAB; and ſome other angle or ſide for proof; and ſet all theſe angles down in the draught. Then by means of theſe, the field may be laid down on paper, and meaſured.

Note, it does not ſignify whether your eye draught be true or falſe, provided all the lines lay but in their right poſition; for the conſtruction at laſt Fig. will make all right. 13.

SCHOLIUM.

As the ſurveying a triangle and a trapezium, is the foundation of ſurveying all other ſorts of figures, I have been very particular in ſhewing different methods. For all plane figures muſt be divided into triangles and trapeziums before they can be meaſured; and therefore the learner ought to have a good notion of theſe.

PROB. XIII.

To ſurvey any field by the chain only.

Let ABDF be the field propoſed. Set up ſtation ſtaves, or any long ſticks, with marks or ſtreamers on them, in every corner of the field; pieces of white or red rags will do well enough, if they can but be ſeen, the breadth of the field. And as you walk over the field, conſider how it may moſt commodiouſly be divided into triangles or trapeziums, ſo as to be eaſieſt meaſured. It muſt be ſo divided, that you may ſee from one end to the other, of any diagonal of a trapezium, or baſe of a triangle; and likewiſe ſee the tops of the perpendiculars, which muſt fall within the figure. Here the field is divided into two trapeziums FCDE and FCBG, and a triangle BGA. Here we might have divided the field into the two trapeziums ABCG and GFEC, and the triangle ECD; and a great many other ways. 14.

Having ſet out the parts of the field, as you deſign to meaſure them; begin at any part as FEDC, and meaſure along the diagonal FD, till you come at the perpendiculars IC and HE, which I have ſhewn how to find in the laſt Prob. there put ſtakes

Fig. in. When the diagonal FD is meaſured, and ſet
14. down, go to E and meaſure EH, and then IC, and
ſet them down; and then you have the firſt trapezium.

Being at C, meaſure along the diagonal CG of the trapezium CFGB, till you come at the perpendiculars LB, KF, to ſet marks, whoſe places are found as before; then meaſure to the end, and ſet down the diagonal; then go to F, and meaſure the perpendicular FK, and then LB, and ſet them down. So you have the ſecond trapezium.

Being at B, meaſure along the baſe BG to the perpendicular NA, and put in a ſtake at N. Then meaſure to G, and ſet down the baſe BG, and laſtly, meaſure the perpendicular AN, and ſet it down; ſo you have the triangle BGA.

The reſult of the work.

1. *Trapezium.*	
The diagonal	11.88
1 perpendicular	3.54
2 perpendicular	2.52
2. *Trapezium.*	
The diagonal	12.00
1 perpendicular	4.47
2 perpendicular	3.72
Triangle.	
Baſe	10.56
Perpendicular	3.32

In meaſuring any of the lines, take care to go very ſtrait; otherwiſe the perpendiculars will be either too ſhort or too long, by going out of the line. But when the perpendiculars are near together, as HE, IC, the going out of the line cauſes little error; becauſe one gains what the other loſes.

When the perpendiculars are not very long, the places where the perpendiculars fall may be taken true enough by the eye, without an index; for a ſmall error in the place, makes no ſenſible difference in the length. Fig. 14.

Otherwiſe thus.

Let ABCDE be a field; from any angle of the field as D (from which all, or moſt of, the other angles may be ſeen), ſuppoſe lines drawn to the other angles, as A and B; which will divide the whole field into triangles; and marks being placed at theſe angles; take an eye draft of the field, thus divided into triangles. Then meaſure all the ſides of every triangle, and ſet them down in the draft, along their reſpective lines. Thus, meaſure DE, EA, AD, in the triangle ADE; then DB, BA, in the triangle ABD, then BC, CD in the triangle BDC. Having got all theſe meaſures, every triangle may be drawn upon paper one by one, and ſo the whole field will be laid down. And your rough draft will inform you, how to join the triangles together. 15.

If the other angles cannot be ſeen from any angle, then you muſt take a point about the middle of the field as F, from which all the angles A, B, C, D, E, can be ſeen. And ſetting up marks at theſe angles, and ſuppoſing lines drawn to them all from F, take a rough draught of the field as uſual. Then meaſure all the ſides of every triangle one by one, and write down their lengths, as you meaſure them, along their correſpondent ſides in the draft. And then they may be all laid down on paper as before. 16.

Note, the fewer triangles there are, the ſooner they are meaſured; and therefore it is ſooneſt done, when the field is divided into triangles from ſome angle, as in the former method. For the leaſt number of triangles, any field can be divided into, is

Fig. 16. leſs by 2 than the number of ſides of the figure, when the diviſions run from an angle; and the diagonals leſs by 3. But when the diviſion is made from a point within; the number of triangles, and number of lines, is equal to the number of ſides.

For proof of the work, ſome line or diagonal, ſhould be meaſured over and above.

If you have not a rough draft, meaſure round the field, AB, BC, CD, &c. and ſet them down in a table. Then meaſure the diagonals AD, BD, which ſet down in a table; or meaſure FA, FB, FC, &c. and ſet down. But take notice what way you go about the field.

PROB. XIV.

To ſurvey any field by the plain table.

15. Set up marks at every corner of the field, ABCDE, except the corner, where you take your ſtation. Therefore chooſe ſuch an angle for your ſtation, from which you can ſee all the other angles. Then put a clean ſheet of paper upon your table and fix it at the angle D, ſo that the needle, if it have one, may hang juſt over the flower-de-luce. Then apply the index to D, and direct the ſights to E, and by the edge of the index, draw the obſcure line DE upon the paper; meaſure DE, and ſet it off (from your ſcale) from D to E. Then turn the index about, till thro' the ſights you can ſee A, the ſide of the index ſtill being at D. Then draw the line DA; meaſure the diſtance, and ſet it from D to A. In like manner direct the ſights to B, C, &c. and draw lines upon your paper; and meaſure the diſtances DB, DC, &c. and ſet them from D to B, C, &c. upon the paper. Laſtly, draw the ſides of the field DE, EA, AB, BC and CD; ſo your draught is finiſhed.

Otherwiſe thus. Fig. 16.

When there is no angle from which all the reſt can be ſeen, take a ſtation in the middle of the field, as at F; from whence all the angles can be ſeen. Then ſet up marks at every corner, and plant the table at F; and turn it about till the needle hangs over the flower-de-luce, and there fix it. Then place the index on the point F, and direct the ſights to A, and draw an obſcure line by the edge of the index upon the paper. Then direct the ſights to B (ſtill keeping the edge of the index cloſe to F), and draw a line. In like manner direct the ſights to C, D and E, and draw lines to C, D, E. Then meaſure all the diſtances, in this order FA, BF, FC, DF, FE; and with your compaſſes ſet the diſtances on their proper lines from F to C, D, E; and laſtly, draw the boundaries of the field AB, BC, CD, DE, EA.

Or thus, by going round the field.

Suppoſe ABCDE a wood or pool of water. Plant the table at A, and chuſing a point on the paper near the edge as A, lay the index upon A, and direct the ſights to E, and draw a black line as AE; then meaſure the diſtance AE, and ſet it from A to E on your paper. Then the table remaining fixt, turn the index about, and direct the ſights from A to B, and draw a line as AB; meaſure the diſtance AB, and ſet from A to B on the paper. 15.

Then remove the table to B, and lay the index upon the line BA; and turn the table about, till thro' the ſights you ſee the mark A. Then fix it, and direct the ſights from B to C, and draw the line BC; then meaſure it, and ſet it from B to C on the paper.

Then remove the table to C, and laying the index on the line CB, turn the table about, till you ſee the mark B; then direct the ſights from C to D, Fig. 15. and draw a line CD on the paper; meaſure CD, and ſet it from C to D. In like manner remove to D and draw DE, and then to E and draw EA. Obſerving at every new ſtation, to lay the index upon the line laſt drawn, and turn the table about, till the index point to the laſt ſtation. For by this means the table is always placed in the very ſame ſituation, at every new ſtation; ſo that all the lines on the table will then be parallel to the reſpective lines in the field. Having thus drawn all the ſides of the field upon your paper, the draft is finiſhed.

Therefore to prove your work, when you are come to any new ſtation, and has fixt the table; turn the index to any former ſtation, or to any angle of the field, or to any known object; and it will paſs thro' the ſame point in the table that repreſents it, if the work is right; and if it do not, there is ſome error committed, which muſt be rectified. Therefore let all your marks ſtand to the laſt, to obſerve by.

Care muſt be taken, that the table be not moved out of its place, till the work at any one ſtation be quite finiſhed. Therefore obſerve at firſt, what object the index points to, when laid upon ſome certain line on your table; for it muſt point to the ſame object at laſt, when laid on the ſame line.

When any very large champion field is thus to be meaſured, it would require the paper to be often changed; to prevent this, uſe the following method. Place the table at every angle, and draw lines as before directed; and meaſure the ſides, but do not ſet the meaſured diſtances upon theſe lines, but only write down the length upon every line. Therefore theſe lines may be drawn as long or as ſhort as you will; and therefore need not run off the table. And when this comes to be laid down, you muſt make all the angles equal to thoſe on the table, and all the ſides of ſuch length as is written upon them, taken from a proper ſcale; ſo you ſhall have the true plan of the field. Fig. 15.

When a field has many ſides, the method of meaſuring round about it, with the plane table, will be ſubject to errors, by reaſon there are ſo many angles to be taken, and always from different lines, which multiplies the error.

PROB. XV.

To ſurvey a field with the infallible.

Firſt, from a point within it, or at any angle, as ſuppoſe at S. Set up marks at all the corners of the field, as at A, B, C, D, &c. and having aſſumed the ſtation S, there fix the inſtrument PQ, with the paper of obſervation upon it. Then direct the index to the angle A, and from the center of the inſtrument draw by its edge the line 1, the ſame way the angle A lies from S. Then meaſure the diſtance SA, and write that diſtance on the line 1. Then direct the index to B, draw the line 2 from the center of the inſtrument, the ſame way that B lies from S; and meaſure the length SB, and write its length on the line 2 upon the paper. In the ſame manner, the inſtrument ſtill remaining fixt, direct the index ſucceſſively to all the angles C, D, E, F, G; and draw as many lines 3, 4, 5, 6, 7, by its edge, upon the paper; and write thereon their reſpective diſtances SC, SD, &c. meaſured with the chain. Thus we find SA 16.20, SB 10.12, SC 14.68, SD 12.90, SE 7.25, SF 8.84, and SG 14.65. Then all the lines 1, 2, 3, &c. will be parallel to the lines SA, SB, SC, &c. of the field. And the work is ready for lying down. And the proceſs had been the ſame, if the ſtation had been in any angle. 17. 18.

Fig. 18. 19.

Otherwiſe, by going round the field.

Place the inſtrument at any angle A, and direct the index to B, and from the center of the paper draw the line 1, the ſame way from the center that B lies from A. Screw the index faſt upon the line 1, and remove to B, and as you go, meaſure AB, and write its length on the line 1. Then fix the inſtrument at B, and turn it round till the index (by backſight) point to A; there ſcrew the table faſt, and direct the index to C, and draw the line 2 by its edge upon the paper, to lie the ſame way from the center, that C lies from B. Then ſcrew faſt the index to the line 2, and remove to C, and as you go along meaſure the ſide BC, and write its length on the line 2. Fix the inſtrument at C, and turn it round till the index point backward to B; then turn the index to D, and by its edge draw the line 3, and ſcrew it faſt. After this manner proceed to all the angles D, E, F, G; always obſerving, to fix the index upon the line laſt drawn, and when you come at the next angle, to fix the inſtrument, that the index may point backward to the laſt ſtation, and then direct the index to the next ſtation; and in going along, meaſure each ſide, and ſet it upon its reſpective line in the paper. And note, at the laſt ſtation G, the index ought to point to A if the work be right. Here AB is 13.10, BC 6.18, CD 17.75, DE 10.32, EF 10.45, FG 9.00, GA 8.15. And all the lines 1, 2, 3, &c. will be parallel to the reſpective lines AB, BC, CD, &c.

SCHOLIUM.

I might now ſhew how to ſurvey a field by the theodelite, circumferentor, ſemicircle, geodetical ſtaff, &c. but as theſe inſtruments are purely for taking angles, the method whereof has been ſhewn in the firſt Problem; therefore no particular di-

rections need be given further for that purpoſe. Fig.
And when theſe inſtruments are fit to be uſed, will 19.
be ſhewn in the following Problems, wherein are explained the different methods of ſurveying any field or parcel of ground.

PROB. XVI.

To ſurvey a Field at one ſtation, by meaſuring ſeveral lines.

Chuſe ſome angle in the field for your ſtation, 20.
from whence you can ſee all the reſt of the angles, as at S. Then ſet up ſtation ſtaves or marks at every corner A, B, C, &c. Then placing the inſtrument at S, whether it be a Theodolite, Semicircle, Geodetical ſtaff, &c. it is the ſame thing; direct the fixt ſights to A, and there fix the inſtrument; then direct the index to B, ſo that thro' the ſights you may ſee the mark at B; and the degrees cut by the index, will ſhew the angle ASB, which ſet down in the field book. Then direct the index to C, as before; and the degrees cut will ſhew the angle ASC, which ſet down. In like manner take the angle ASD, and the reſt, if there be more, and ſet all down. And this way is more exact, than taking the angles ASB, BSC, CSD, &c.

Then with the chain meaſure the ſides and diſtances in this order SA, BS, SC, DS; ſetting them down in the field book, or writing them on the lines in a rude draft of the field.

Or thus, from a point within.

When all the angles cannot be ſeen from any one 21.
of the angles of the field; aſſume ſome ſtation S within the field, where the angles can all be ſeen; there place the inſtrument at S, and directing the fixt ſights to A, there fix the inſtrument, and direct the index to B, and turning it round, till thro' the ſights you ſee the mark at B, obſerve the degrees

Fig. degrees cut by the index, for that is the angle ASB,
21. which ſet down in the field book, or in a rude draft of the field. Then the inſtrument remaining fixt, direct the index to C, and the degrees cut, ſhew the angle ASC. In the ſame manner direct the index to all the other corners D, E of the field; and you will get all the angles about S, meaſured from the firſt line SA, which ſet down. And this is more exact, than taking the angles ASB, BSC, &c. ſingly. But with ſome inſtruments you cannot take the angles quite round from the line SA; and therefore you muſt then take the angles ſeparate, ASB, BSC, CSD, DSE and ESA. And to prove the work, the ſum of all theſe angles ought to be equal to 360 degrees. But if the ſtation S happens to be upon an eminence, the ſum of all theſe angles will be ſomething leſs than 360 degrees.

Having got the angles, meaſure all the diſtances SA, SB, SC, &c. with the chain. And it is quickeſt to meaſure in this order, SA, BS, SC, DS, SE; that is, from the inſtrument to the firſt corner, and back from the ſecond corner to the inſtrument, and ſo on. Still one from the inſtrument, and another to it. Then theſe diſtances muſt all be ſet down in your book or draft.

Otherwiſe, by meaſuring round the field.

22. Take your ſtation, if you can, in an angle or a ſide; let it be at E. Then marks being placed at A, B, C, D; and turning the fixt ſights to A, fix the inſtrument; and direct the index to B, and the degrees cut will ſhew the angle BEA, which note down. Then turn the index to C, and then to D, and note down the degrees cut, which are the angles AEC, AED.

Then meaſure with the chain all the ſides EA, AB, BC, CD, and DE; and ſet them down.

Here if you take all the angles at E ſeparate, the ſum of them all muſt be equal to the angle AED, if the work be right. You muſt note whether the angles at B, C, E, be acute or obtuſe. Fig. 22.

In all theſe methods, care muſt be taken that the inſtrument be not moved out of its place, while you are working; for if it be, your angles will not be true.

PROB. XVII.

To ſurvey a field, by meaſuring but one line, and taking the angles round about.

Let ADF be the field; ſet marks up at all the corners F, A, B, C, D, E; and chuſe ſome ſide, or ſome diagonal or line, to be meaſured, ſuppoſe AF. Then place the inſtrument at F, and take the angles AFB, BFC, CFD, DFE; and note them down. Go to A, and take the angle FAB. In like manner going round the field, take the angles FBC, FCD, FDE; laſtly meaſure FA. Then theſe triangles may be laid down upon paper, one after another, beginning at the triangle FAB. 23.

In like manner any other line, as FB or FC, may be meaſured inſtead of FA; and then the triangle FBC or FCD muſt begin the operation.

Or thus, from a ſtation in the field.

Aſſume the ſtation S, and let FS be meaſured. Place the inſtrument at S, and take all the angles round S, ASB, BSC, CSD, DSE, ESF, and FSA. Then go round the field, and take the angles SAB, SBC, SCD, SDE, SEF, and SFA. From theſe, the field may be laid down, beginning at the triangle SFA. 24.

And any line AS, BS, &c. may be meaſured, inſtead of FS; if AS be meaſured, then the conſtruction muſt begin with the triangle ASB.

Theſe

Fig. Theſe methods are ſubject to errors, eſpecially when the number of ſides is very great.

PROB. XVIII.

To ſurvey a field, by the method of circulation, or going round it.

25. Let ABCDE be the field. In this caſe we muſt reduce the field to as few ſides as we can; and therefore where there are any ſmall bendings, as in crooked hedges, we muſt take offsets by Prob. III. Therefore pitch upon the ſtations, *a*, *b*, *c*, *d*, *e*, as near the hedges as you can conveniently come; and at theſe places ſet up ſtation ſtaves with marks. Begin at ſome angle as *a*, and placing the inſtrument there, take the angle *eab*, which ſet down in the field book. Then meaſure from *a* to *b*; and in going, take the offsets at *a*, and at *f*, *g*, *h*, *i*, and at *b*, and note them down in the field book, and the length *ab*. Then place the inſtrument at *b*, and take the angle *c*, and note it down; and then meaſure from *b* to *c*; and in your way, take the offset at *b*, and where you touch the hedge at *k*, and the offset at *c*; and ſet them down. Then at *c* place your inſtrument, and take the angle *bcd*, which write down; and then meaſure *cd*, taking the offsets at *c* and *d*, which write down. Again place the inſtrument at *d*, and take the angle *cde*, and ſet it down, and the offset at *d*. Then meaſure from *d* to *e*; and as you go, note the point *l* where you touch the hedge, and the offsets at *m* and *e*; and ſet all down. Then ſet the inſtrument at *e*, and take the angle *dea*, and ſet it down, take the offset at *e*, and meaſure from *e* to *a*, being a ſtreight hedge, and take the offset at *a*. Set all down, and your work is finiſhed.

In your book you muſt note which are inward, Fig. and which are outward angles. And to prove your 25. work, add all the angles together, and the ſum will make twice as many right angles, abating 4, as the field has ſides, if the work be right.

When there are many angles or ſides in a field, this method is ſubject to errors; therefore it ſhould not be applied, where there are more than 6 ſides; or at leaſt the field ſhould be reduced to that number; for the fewer ſides, the more exact the work will be. For when the number of ſides is very great, the error is multiplied with the number of ſides, becauſe every angle is taken from a different line; and for that reaſon the draught will never cloſe. For when all is laid down the laſt point ſhould fall upon the firſt, for we end where we begun; and when the plan does not anſwer that, it muſt be wrong.

When you cannot get an internal angle conveniently, produce one of the ſides without the field, and take the external angle. And if all the angles be taken externally, the ſum of them all ought to be equal to 4 right angles. And if a ſide cannot be meaſured directly, it may be done by the method of taking inacceſſible diſtances, in the laſt Section.

If you make uſe of the circumferentor for taking angles, you need only place it at every other angle. And the like for any inſtrument, where a needle is made uſe of.

Otherwiſe, by the needle.

When the number of angles of any piece of ground is very great, the method of circulation, by taking the angles round it, is impracticable, becauſe there is no fixt line to take theſe angles from. In this caſe the needle may be brought in, to our aſſiſtance. And here we have no more to do, Fig. do, but to go from angle to angle, and take the bearings of all the ſides, or the angles they make with the meridian; and to meaſure the lengths of the ſides.

26. Let ABCDEFGH be ſome thick wood or pool of water, which we cannot ſurvey within. Here we muſt take all the boundaries on the outſide, and reduce it to as few ſides as we can, for greater exactneſs; as in this field, where we have but 5 ſtations; but then we ſhall have the more offsets to be meaſured. Therefore having ſet up marks at *a*, *b*, *c*, *d*, *e*, where we take our ſtations; begin at any one as *a*, and placing the compaſs and needle there, ſo that the needle may hang over the flower-de-luce, take the bearing of the line *ab*, and ſet down what angle it makes with the meridian, and which way it lies in reſpect of E. W. N. or S. Then meaſure the length *ab* to the next ſtation, and in going, take the offsets at *a*, *s*, *t*, and where it touches the corner at B; and ſet all down. Then place the compaſs at *b*, and cauſe the needle to hang over the flower-de-luce; there fix it, and take the bearing, of the next ſtation *c*, or of the line *bc*; and then meaſure *bc*, and then take the offsets at *b*, *f*, *h*, *i*, and where it touches the corner at C; and ſet theſe down. Then place the compaſs at *c*, *d*, and *e* ſucceſſively; and take the bearings of *cd*, *de*, *ea*, as before; and the offsets at *k*, E, *d*, and at *m*, *n*, F, *p*, G; alſo at the corners H and A; and ſet them all down in the field book, and the work is done.

Care muſt by taken, that there be no iron near the needle, for that will make it deviate from the meridian. And the inſtrument need only be placed at every other angle, but then you muſt take the bearings both ways, that is, of both ſides of the incloſed angle, the one forward, the other backward.

When the ſurvey is done, if you lay down every Fig. 26. diſtance ſingly, and the bearing, upon paper; and from thence get the eaſting, weſting, northing or ſouthing, and put them into diſtinct columns; then the ſum of the eaſtings and of the weſtings muſt be equal. And the ſum of the northings equal to the ſum of the ſouthings; if the work be right. For it is no more than working a traverſe on the land, by plain ſailing, as it is practiſed at ſea.

Tho' this method is not to be relied on, or uſed, but in caſes of neceſſity, where there is a great number of angles to be taken, by reaſon one cannot take an angle with the needle, to any great exactneſs; yet there is no greater error in taking one angle, than there is in taking 20, becauſe they are all taken from the meridian, and therefore are independent of one another. But if it was poſſible to take an angle as exactly with the needle as with another inſtrument, it would be the beſt inſtrument to ſurvey with. And as it is, it has the advantage of other inſtruments in ſuch caſes where the number of ſides is very great. The main difficulty is, to fix the inſtrument right by help of the needle, by which it is to be placed truly in the meridian; and to help this, the needle muſt be made very long, and be well touched. In uſing other inſtruments, they are fixt, by directing the index to the laſt ſtation, which admits of greater exactneſs. For all inſtruments with a needle, and alſo the plane table, whether it has a needle or not, require at every ſtation to be placed in the ſame poſition, in reſpect to the meridian, thro' all the operations, which is hard to do; for a needle, tho' never ſo good, will not always ſtand at the very ſame point.

PROB. XIX.

To ſurvey a field at two ſtations meaſuring only one line, which is the ſtationary diſtance.

27 Chooſe two ſtations S and T, in the field, from whence all the angles can be ſeen. Then ſet up ſtation ſtaves there, and at every angle, with ſtreamers of different colours on them. Then place the inſtrument at one ſtation S, and directing the fixt ſights to the other T, there fix the inſtrument; then directing the index ſucceſſively to the ſeveral angles in order, take the angles TSA, TSB, TSC, TSD, TSE, TSF; and number them 1, 2, 3, 4, &c. for diſtinction, and write them all down. And the marks at theſe different angles, ought all to be different, to diſtinguiſh them one from another. Then go to the ſecond ſtation T, and directing the fixt ſights to the firſt ſtation S, fix it there; and directing the index to the ſeveral angles, in the ſame order as before, take the angles STA, STB, STC, STD, STE, STF; and number them, and ſet them all down as before. To take the angles in the ſame order, you ought to know them from one another by the marks; for if you do not know which is which, all your work will be confuſion. And as you go from one ſtation to the other, meaſure the ſtationary diſtance ST. Then if all theſe angles be laid down upon paper, and lines drawn, the correſpondent lines will interſect in the angles or corners of the field A, B, C, &c. Then draw the lines AB, BC, CD, &c.

If the plane table is uſed.

Draw the ſtationary line ST on the table, and ſetting the index to the line ST, direct it to T, and fix the table there; and laying the edge of the index

Fig. 27. to S, direct it ſucceſſively to all the angles A, B, C, &c. and draw lines from S by the edge of the index. Then go to T, and meaſure the ſtationary diſtance ST, which ſet off upon the table. Lay the index upon the line ST, and direct it to S, and fix the table ſo. Then lay the edge of the index to T, and direct it ſucceſſively to the ſame angles A, B, C, &c. and draw lines from T as before; and note where the correſpondent lines (or thoſe drawn to the ſame angle), interſect; for theſe will be the ſeveral angles of the field.

28. The two ſtations S and T may be ſometimes taken in one of the ſides, as in FE; and then there will be fewer angles to take. Or they may be taken out of the field, as at *s* and *t*, as when one is not admitted to come into the ground. And it will ſhorten the work, if one or both ſtations are taken in the production of the ſides of the figure, as here, in AF and DE; and likewiſe if two of the angles fall in the ſame right line, drawn from either ſtation.

Note, the corners of the field thus determined by the interſection of lines, will be trueſt, when they interſect the neareſt to right angles. But when they cut very obliquely as at A, ſuch interſection will not give the point exactly; and therefore it would be better to meaſure one of the lines, as SA or TA. And the error of interſection will be reciprocally as the ſine of the angle SAT, *ceteris paribus*.

If it happen that ſome angle cannot be ſeen from one of the ſtations; then ſet up a ſtake for a mark, in the midway between the ſtation and that angle, to ſtand in a right line; which may ſoon be done by an inſtrument, placed at the ſtake, taking backſight and foreſight. Or take a new ſtation, ſo that the angle or angles wanting, may be ſeen from this and one of the others, and proceed as before.

Fig. 28. Thus a map of the chief places in a town may be made; or any part of a river ſurveyed, &c. by taking two ſtations, on two towers, or two high hills; and thus inacceſſible places may be meaſured by means of ſcaffolds erected, and fixing high poles at the angles, with flags of different colours flying in the air.

But in ſuch caſes, it is beſt to have but a few poles up at once, as half a dozen; and when the places of theſe are determined on the draught, fix them up in freſh places; and ſo as many places as you will, may be determined, by taking a few at a time, and repeating the ſame work.

In uſing the plain table, it ſometime happens that the lines run off the table; in this caſe, move the paper ſo far off the table, that the laſt ſtation may be a little way upon it. Then take a freſh ſheet of paper and glue it to the other, at that ſide with mouth glue, and ſo faſten them down. Then lay a ruler upon that ſtation and the ſtationary line, and prolong it upon the freſh paper. Then lay the index upon the ſtationary line, and turn the table, till thro' the ſights you ſee the laſt ſtation; there fix it, and proceed.

PROB. XX.

By means of ſeveral ſtations, to ſurvey a large common, a river, a part of a country, or any hilly ground.

When all the angles of a field or parcel of ground, or all the objects we want to obſerve, cannot be ſeen from one place; then we muſt take ſo many ſtations, as will ſucceſſively diſcover them all to us.

29. Let ADG be a large common, or part of a country; and A, B, C, D, E, &c. the angles or boundaries of it, or any remarkable objects in the country;

country, as trees, towers, houſes, mills, &c. Take the ſtations S, T, V, &c. upon ſome hills or eminent places, from which all the angles or objects can be ſeen, and at all the places and ſtations, ſet up long poles with marks. Suppoſe the objects K, L, A, B, C can be ſeen from S; place the inſtrument at S the firſt ſtation, and direct the fixt ſights to the ſecond ſtation T, and there fix the inſtrument. Then take all the angles round S, as TSK, TSL, TSA, TSB, TSC; and meaſure all the diſtances SK, SL, SA, SB, SC, and the ſtationary diſtance ST; and ſet all down. But in the plane table, fix the table and direct the index from S to T, and draw the ſtationary diſtance ST. Then direct ſucceſſively from S to K, L, A, B, C; and draw lines on the table. Then meaſure the diſtances SK, SL, SA, SB, SC, and ST, and ſet them all upon their proper lines. Fig. 29.

Then remove to your ſecond ſtation T, where you can ſee the objects D, E, F, I. Direct the fixt ſights to the former ſtation S, and fix the inſtrument; then take all the angles from ST, as STD, STE, STF, STI; and to the next ſtation STV, and meaſure the diſtances TD, TE, TF, TI, and TV, and ſet all down. Or in the plane table, lay the index on the ſtationary line ST, and keeping it there, turn the table about, till thro' the ſights you ſee the former ſtation S, there fix it. Then direct the index to all the places D, E, F, I, and V; and draw lines, on which ſet the diſtances as before.

Then go to a third ſtation V, where you can ſee the objects G, H. Then placing the inſtrument at V, direct the fixt ſights to T the former ſtation, and fix the inſtrument, and take all the angles from TV, as TVG, TVH, and to the next ſtation, if there be any more; and meaſure

Fig. 29. the diſtances, VG, VH; and ſet all down. But in the plane table, lay the index on the laſt ſtationary line VT, and turn the table about till the former ſtation T appear thro' the ſights, there fix it. Then direct the index ſucceſſively to G and H, and draw lines, on which ſet the meaſured diſtances, VG, VH as before.

And in general when any ſtationary line is drawn on the table, and you come to a new ſtation, lay the index upon that ſtationary line, and turn the table, till thro' the ſights you ſee the ſtation laſt come from, and there ſcrew it faſt, and proceed.

And after this manner, if there be more objects undetermined, you muſt take new ſtations, and ſo proceed, till all is finiſhed.

Then from all theſe angles and diſtances given, the whole work is eaſily laid down by rule and compaſs, and the plan or map deſcribed. But when it is taken by the plain table, it is deſcribed already, by the operations on the table. And if it be a field, lines muſt be drawn from A to B, from B to C, &c.

At every ſtation when the inſtrument is fixt, take care it be not moved out of its place, till all the work at that ſtation be done; which you may know by obſerving ſome remarkable object thro' the fixt ſights; for that object ought always to appear there. Or on the table by drawing a line; or making a mark, by the edge of the index. For then the index being laid to that mark, that object ought always to appear thro' the ſights.

If there be ſome of the objects which cannot be ſeen from any of the former ſtations, then new ſtations muſt be taken from which they can be ſeen. Remembring to take their diſtances, and angles (or poſitions), from ſome of the other ſtations.

If ſome of the objects can be ſeen from two adjoining ſtations, as I from both T and V, then the angles may be taken from both ſtations (ITV and IVT); which will ſave the labour of meaſuring to ſuch objects, the diſtances TI, VI. Fig. 29.

In the plain table, when any ſtation line runs off the table, take off the paper and glue another paper to it with mouth glue; on this produce that ſtation line to its proper length. Then faſten the paper on the table, with the former ſtation near the edge of the table; and laying the index on the new ſtationary line, turn the table about, till you ſee the laſt ſtation come from, thro' the ſights; there fix it and proceed.

It will be proper always to take an eye draught of the field or country, upon paper, before you begin your work. And all your meaſures muſt be written down in this rough draught. Or elſe they muſt be written regularly in a field book on purpoſe.

PROB. XXI.

To ſurvey a lordſhip, or large eſtate of land.

If the eſtate be very large, and contains a great number of fields; it cannot be done by ſurveying all the fields ſingly, and then putting them together; nor can it be done by taking all the angles and boundaries that incloſe it. For in theſe caſes, any ſmall errors will be ſo multiplied as to render it very much diſtorted.

1. Walk over the lordſhip two or three times, in order to get a perfect idea of it, and till you can carry the map of it in your head. And to help your memory, draw an eye draught of it on paper, to guide you, or at leaſt, of the principal parts of it.

2. Chooſe

Fig. 29. 2. Chooſe two or more eminent places in the eſtate, for your ſtations, from whence you can ſee all the principal parts of it; and the fewer ſtations you have to command the whole, the more exact your work will be; and let theſe ſtations be as far diſtant from one another as poſſible; and they will be fitter for your purpoſe, if theſe ſtationary lines be in or near the boundaries of the ground, and eſpecially if two lines or more proceed from one ſtation.

3. Take what angles, between the ſtations, you think neceſſary, and meaſure the diſtances from ſtation to ſtation, always in a right line; theſe things muſt be done, till you get as many angles and lines as are ſufficient for determining all your points of ſtation. And in meaſuring any of theſe ſtationary diſtances, mark accurately where theſe lines meet with any hedges, ditches, roads, lanes, paths, rivulets, &c. and where any remarkable object is placed, by meaſuring its diſtance from the ſtationary line; and where a perpendicular from it cuts that line. And always mind, in any of theſe obſervations, that you be in a right line, which you will know by taking backſight and foreſight, along your ſtationary line; which you muſt never omit. And thus as you go along any main ſtationary line, take offsets to the ends of all hedges, and to any pond, houſe, mill, bridge, &c omitting nothing that is remarkable, and all theſe things muſt be noted down, for theſe are your *data*, by which the places of ſuch objects are to be determined upon your plan. And be ſure to ſet marks up at the interſections of all hedges with the ſtationary line, that you may know where to meaſure from, when you come to ſurvey theſe particular fields, which muſt immediately be done, as ſoon as you have meaſured that ſtationary line, whilſt they are freſh in memory. By this means all your ſtationary lines are to be

meaſured, and the ſituation of all places adjoining to them determined, which is the firſt grand point to be obtained. I would have you lay down your work upon paper every night, when you go home, that you may ſee how you go on.

4. As to the inner parts of the eſtate, they muſt be determined in like manner, by new ſtationary lines. For after the main ſtations are determined, and every thing adjoining to them; then the eſtate muſt be ſubdivided into two or three parts by new ſtationary lines; taking inner ſtations at proper places, where you can have the beſt view; and meaſure theſe ſtationary lines as you did the firſt, and all their interſections with hedges, and all off-ſets to ſuch objects as appear; then you may proceed to ſurvey the adjoining fields, by taking the angles that the ſides make with the ſtationary line, at the interſections, and meaſuring the diſtances to each corner, from the interſections. For every ſtationary line will be a *baſis* to all the future operations; the ſituation of all parts being entirely dependent thereon; and therefore they ſhould be taken as long as poſſible; and are beſt to run along ſome of the hedges or boundaries of one or more fields, or to paſs through ſome of their angles. All things being determined for theſe ſtations, you muſt take more inner ſtations, and continue to divide and ſubdivide; till at laſt you come to ſingle fields; repeating the ſame work for the inner ſtations, as for the outer ones, till all be done. And cloſe the work as oft as you can, and in as few lines as poſſible. And as it may require ſome judgment to chuſe ſtations the moſt conveniently, ſo as to cauſe the leaſt labour; let the ſtationary lines run as far as you can along ſome hedges, thro' as many corners of the fields, and other remarkable points, as you can. And take notice how one field lies by ano-

Fig. ther; that you may not miſplace them in the draught.

5. An eſtate may be ſo ſituated, that the whole cannot be ſurveyed together; becauſe one part of the eſtate cannot be ſeen from another. In this caſe, you may divide it into three or four parts, and ſurvey the parts ſeparately, as if they were lands belonging to different perſons; and at laſt join them together.

6. As it is neceſſary to protact or lay down your work as you proceed in it, you muſt have a ſcale of a due length to do it by. To get ſuch a ſcale, you muſt meaſure the whole length of the eſtate in chains; then you muſt conſider how many inches long the map is to be; and from theſe you will know how many chains you muſt have in an inch, and make your ſcale, or chuſe one already made, accordingly.

7. The trees in every hedge row muſt be placed in their proper ſituation, which is ſoon done by the plane table; but may be done by the eye without an inſtrument; and being thus taken by gueſs, in a rough draught, they will be exact enough, being only to look at; except it be ſuch as are at any remarkable places, as at the ends of hedges, at ſtiles, gates, &c. and theſe muſt be meaſured. But all this need not be done till the draught be finiſhed. And obſerve in all the hedges, what ſide the gutter is on, and to whom the fences belong.

8. When you have long ſtations, you ought to have a good inſtrument to take angles with, which ſhould be exact to a quarter of a degree at leaſt; and hardly any common ſurveying inſtrument will come nearer. And tho' the plane table is not at all a proper inſtrument to ſurvey a whole lordſhip with; yet it may very properly be made uſe of, to take the ſeveral ſmall internal parts; and ſuch as cannot

be taken from the main ſtations; and is a very quick and ready inſtrument. Fig.

Example.

Walking over the lordſhip, I pitch upon the four ſtations A, B, C, D, from which I can command the greateſt part of it, there I ſet up marks. Then I meaſure along AB, which is a right line, and the boundary on one ſide of the land. In meaſuring along, I ſet down the diſtances meaſured, when I come at the corners of the fields *a*, *a*, *a*, *a*, where the hedges come in, and likewiſe where I croſs the brook *bb*. Then having got to B, I ſet down the whole length of AB. 30.

Next I meaſure from B to C, and in my way, I ſet down how far I have meaſured when I croſs the hedges at *c*, *c*, *c*, *c*; and likewiſe where I croſs the brook *bb* again. Thus I meaſure forward till I come at C, and then I ſet down the length of the ſtationary line BC.

After the ſame manner I meaſure along the ſtationary line CA, obſerving to ſet down the interſections with the hedges, as before; till I come at A, where I ſet down the length of CA. Then the three points A, B and C, are determined; and may be laid down in the plan; and all the foreſaid points.

Being come to A again, I go from A towards B, and in my way I ſurvey every ſingle field adjoining to the ſtationary line AB. To do which the ſhorteſt way, I take the angles at every interſection *a*, that the ſides of each field makes with the ſtationary line AB; and then I meaſure their lengths; by which every field is eaſily laid down. In the ſame manner I proceed from B to C, and meaſure every field adjoining to BC. And then I go to A, and meaſure every field in my way thither.

Next

Fig. 30. Next I go from A towards D, and ſet down as before, all my croſſing of the hedges; and the length AD, when I come at D. And in like manner, I meaſure along DC, ſetting down all the croſſings of the hedges as before, with whatever elſe is remarkable, as where a highway croſſes at *d*.

Having finiſhed all the main ſtations, we muſt begin to make inner ſtations. Therefore I take F and G for two ſtations, being in the lines AB and BC, the hedges from F to G running almoſt ſtreight; then I meaſure from F towards G, and at *f*, I find a hedge going to the left, and going on to *g*, I find another hedge going to the right; and at *h* I croſs the burn. At *i* there is an angle, to which I make an offset. Going on further I come at a croſs hedge *l* going to the right; and then meaſure on to G the end of the ſtation. Now in going from F to G, we can take all the angles that the ſides of the fields make with the ſtationary line FG, and then meaſure their lengths; by which theſe fields may be laid down on paper.

Then I take another inner ſtation at I, and meaſuring from A to *o*, I come to the oppoſite corner of the field; then meaſuring on to *p*, I croſs a hedge; then I proceed to my ſtation I. Then I meaſure from I to F, and take an offset to *n* where the hedge croſſes the brook. Then I come to the corner of the laſt field at *m*; and then meaſure to the oppoſite corner at F, the other ſtation. In your going from A to I, you may take the angles that the hedges make with your ſtationary line AI, and meaſure theſe hedges, and then they may be laid down. And the like, in going from I to F.

All this being done, take a new ſtation H, and meaſuring from B towards H, all the hedges lie almoſt in a right line. So going along we come at a croſs hedge, and going further we come at a tree, in the hedge we meaſure along; going further we come at two other croſs hedges; and a piece further we croſs the brook, going on we come at a croſs hedge; going on ſtill we come at another croſs hedge; all theſe hedges are to the left. Then going on ſtill further, we have a windmill to the right; and afterwards a croſs hedge to the left, and then we meaſure on to the ſtation H. Then meaſuring from H towards C, we have a houſe on the left; and then go on to C. And the fields may be all ſurveyed as you go along BH and HC, and then laid down. And after this manner you muſt proceed thro' the whole, taking new ſtations, till all be done. Fig. 30.

PROB. XXII.

To ſurvey a whole county, or large tract of land.

1. Chuſe two, three or four eminent places for ſtations, ſuch as the tops of high hills or mountains, towers, or church ſteeples, which may be ſeen from one another; and from which moſt of the towns, and other places of note, may alſo be ſeen. And let them be as far diſtant from one another as poſſible. Upon theſe places raiſe beacons, or long poles, with flags of different colours flying at them; ſo as to be viſible from all the other ſtations.

2. At all the places, which you would put down in the map, ſet up long poles with flags at them of ſeveral colours, to diſtinguiſh the places from one another; fixing them upon the tops of church ſteeples, or the tops of houſes, or in the centers of leſſer towns. But you need not have theſe marks at many places at once, as ſuppoſe half a ſcore at a time. For when the angles have been taken, at the two ſtations, to all theſe places, the marks may be moved to new places; and ſo ſucceſſively to all the places you want. Theſe marks then being ſet up at a convenient number of places, and ſuch as may

Fig. be ſeen from both ſtations; go to one of theſe ſtations; and with an inſtrument to take angles, ſtanding at that ſtation, take all the angles between the other ſtation, and each of theſe marks, obſerving which is blue, which red, &c. and which hand they lie on; and ſet all down with their colours. Then go to the other ſtation, and take all the angles between the firſt ſtation, and each of the former marks, and ſet them down with the others; each againſt his fellow with the ſame colour. You may if you can, alſo take the angles at ſome third ſtation, which may ſerve to prove the work, if the three lines interſect in that point, where any mark ſtands. The marks muſt ſtand till the obſervations are finiſhed at both ſtations; and then they muſt be taken down, and ſet up at freſh places. And the ſame operations muſt be performed, at both ſtations, for theſe freſh places; and the like for others. Your inſtrument for taking angles, muſt be an exceeding good one, made on purpoſe with teleſcopic ſights; and of three, four or five feet radius. A circumferentor is reckoned a good inſtrument for this purpoſe.

3. And tho' it is not abſolutely neceſſary to meaſure any diſtance, becauſe any ſtationary line being laid down from any ſcale, all the other lines will be proportional thereto. Yet ſome of the lines had better be meaſured, to aſcertain the diſtances of places in miles; and to know how many geometrical miles there are in any length; and from thence to make a ſcale to meaſure any diſtance in miles. In meaſuring any diſtance, it will not be exact enough to go along the high roads, by reaſon of their turnings and windings; and hardly ever lying in a right line between the ſtations, which muſt cauſe infinite reductions, and create endleſs trouble to make it a right line; for which reaſon it can never be exact. But a better way is to meaſure in a right line with a chain, between ſtation and ſtation,

over hills and dales or level fields, and all obſtacles. Only in caſe of water, woods, towns, crags, rocks, banks, &c. where one cannot paſs; ſuch parts of the line muſt be meaſured by the methods of inacceſſible diſtances; and beſides, allowing for aſcents and deſcents, when we meet with them. And a good compaſs that ſhews the bearing of the two ſtations, will always direct you to go ſtreight, when you do not ſee the two ſtations; and in your progreſs, if you can go ſtreight, you may take offſets to any remarkable places. Likewiſe note the interſection of your ſtationary line with all roads, rivers, &c.

4. And from all your ſtations, and in your whole progreſs, be very particular in obſerving ſea coaſts, rivers mouths, towns, caſtles, houſes, churches, wind-mills, water-mills, trees, rocks, ſands, roads, bridges, fords, ferries, woods, hills, mountains, rills, brooks, parks, beacons, ſluices, floodgates, locks, &c. and in general all remarks or rarities.

5. After you have done with your firſt and main ſtationary lines, which command the whole county; then you muſt take inner ſtations, at ſome places already determined, which will divide the whole into ſeveral partitions: and from theſe ſtations you muſt determine the places of as many of the remaining towns as you can. And if any remain in that part, you muſt take more ſtations, at ſome places already determined; from which you may determine the reſt. And thus we muſt go thro' all the parts of the county; taking ſtation after ſtation, till we have determined all we want. And in general, the ſtationary diſtances muſt always paſs thro' ſuch remarkable points as have been determined before, by the former ſtations.

6. Laſtly, the poſition of the ſtationary line you meaſure, or the point of the compaſs it lies on, muſt be determined by aſtronomical obſervation.

Hang

Fig. Hang up a thread and plummet in the ſun, and obſerve when the ſhadow runs along that ſtationary line; and at that moment take the ſun's altitude; then having his declination, and the latitude; the azimuth will be found, by caſe 11th of oblique ſpherical triangles. And the azimuth is the angle the ſtationary line makes with the meridian; and therefore a meridian may eaſily be drawn thro' the map.

Or a meridian may be drawn thro' it by hanging up two threads in a line with the pole ſtar, when he is juſt north, which may be known from the aſtronomical tables. Or thus, obſerve the ſtar *Allioth*, or that in the rump of the great bear (being that next the ſquare); or elſe *Caſſiopeia*'s *hip*; I ſay, obſerve by a line and plummet when either of theſe ſtars and the pole ſtar come into a perpendicular; and at that time they are full north. Therefore two perpendicular lines being fixt at that moment, towards theſe two ſtars, will give the poſition of the meridian.

County ſurveying is by far the moſt difficult part of ſurveying; for the ſtations are ſo long and ſo many, that if the inſtrument to take the angles, be not very accurate, great errors will be committed; eſpecially after ſo many repetitions of taking angles, at ſo many different ſtations. And to meaſure all the roads and diſtances of places, would be an endleſs labour, and beſides they would not be exact; for no road leading from one place to another, goes in a right line; ſo that any one of them, muſt meaſure to more conſiderably than what it is. Therefore I cannot find that any better method can be practiſed, than what is here laid down. For if a ſtationary diſtance, ſufficiently long be ſet out and meaſured for a baſe; and the angles taken from theſe two ſtations, to all the places that can be ſeen; the ſituation of theſe places will be had more exactly than by any other method. But when we go to theſe ſecond ſtations to take angles, more care is required; and more ſtill, if we go to third, or fourth ſtations, or farther. So that we had need to have good inſtruments to perform this. All this is a work of time and expence. Fig.

PROB. XXIII.

To ſurvey a city or great town.

Suppoſe ABCDEFG be ſeveral ſtreets in any town or city, which you want to ſurvey. Begin at the meeting of two or more of ſome of the principal ſtreets, along which you can have the longeſt proſpect, for getting the longeſt ſtationary diſtances. Let B be the firſt ſtation; then along theſe ſtreets BC and BF, as far as you can ſee, cauſe two men to ſtand for marks; or rather ſet up ſtation ſtaves in wood pedeſtals. Then with your inſtrument placed at B, take the angle CBF, which note down. Then let the mark at F be carried to B, and the mark at C to D. Then you muſt obſerve at B a ſtreet running off to the right; and going along BC, meaſuring with a chain of 50 feet long, you come to a ſtreet at *b* on the left hand, ſet that down, and how far you have meaſured; and proceeding on to C, ſet down the length of BC in your book of obſervations. 31.

Then planting the inſtrument at C, take the angle BCD, and ſet it down. Then let the mark at B be brought to C, and that at D to E, and obſerve at C a ſtreet running off to the right; then meaſure along from C to D; and at *m* you come to two ſtreets going off, one to the left, the other to the right; then meaſure along to D, and ſet all down.

Plant

Fig. Plant the inſtrument at D, and take the angle
31. CDE, and meaſure along from D to E, and at E you have a ſtreet going to the right, and ſet all down. Bring the marks from C to D, and from E to F.

Place the inſtrument at E, and take the angle DEF, which ſet down. Then carry the marks from D to E, and from F to B; and meaſure along EF, and you come to two ſtreets at *n*, one on the right, the other on the left, which ſet down; then meaſure to F, where you have a ſtreet going to the right; ſet all down.

Laſtly, place the inſtrument at F, and take the angle EFB, which ſet down; and at F you have a ſtreet going to the right. Meaſure from F to B, and in your way you have two ſtreets at *p*, one on the right, and the other on the left; ſet them down and proceed to B; and ſet down the length FB, and then you arrive at B where you begun; ſo that part of the ſurvey is finiſhed.

After the ſame manner the part ABFG muſt be ſurveyed, taking the angles at A and B, at F and G; and meaſuring the diſtances AB, BF, FG and GA. And ſo you muſt proceed piece by piece, by this method of circulation, till the whole be finiſhed. And all the parts muſt at laſt be joined together, in their true ſituations.

Obſerve in every part to take in as much of the town as you can at a time; and to cloſe (that is, to come round to the ſame point again) with the feweſt ſides or ſtations you can, not exceeding 5 or 6, if you can help it; for the more ſides the leſs exact. Your chain muſt be 50 feet long, a link to a foot.

At every ſtation, and at other places where it is neceſſary, meaſure the breadth of the ſtreets, taking them as offsets; and as you go along, take offſets to all remarkable places, on the right and left, and to all ends of ſtreets, and to all turnings and corners; and this muſt be done with a rod 10 feet long divided into feet; for all theſe meaſures muſt be taken, and laid down in feet; and ſet all down regularly in your ſurvey book. And to help your memory, it will be convenient to have an eye draught of each part of the town, as you ſurvey it; and always take notice what way you go about. Fig. 37.

Having thus taken your high ſtreets, you may in the ſame manner proceed with the other ſmall ſtreets and lanes, noting down in your book, or eye draught, the offsets, as you find them to the right or left.

Your high ſtreets and croſs ſtreets being taken, you muſt take the meaſures of the houſes to the front, and the breadth of them, with your rod; and the dimenſions of courts and alleys, ſetting them down in your draught. You muſt be very exact and curious in taking the dimenſions of churches, the ſteeple, the buttreſſes, &c. And likewiſe halls, inns of court, colleges, eminent houſes, &c.

SECT. II.

Of Planning or Plotting of Ground; and of Caſting up, Dividing, Reducing, &c.

PROB. I.

To make a ſcale to lay down any plan with.

32. FIRST conſider the bigneſs of your ſcale, or how many equal parts you will have in an inch. Then draw the indefinite line AB, and two more parallel thereto, the one very near. Then take what length you deſign for 10 in your compaſſes; run over the line AB, with the compaſſes, as many of theſe lengths as you pleaſe. Divide the firſt part at A into 10 equal parts; and at all the other points of diviſion, write 0, 10, 20, 30, 40, &c. continued as far as you have occaſion for. And at the ends of the ſcale, and at all the points of diviſion, draw perpendiculars to AB, thro' all the three parallels, and the ſcale is finiſhed.

The uſe is this; take the length of any line you want, to lay down, off the ſcale in your compaſſes, and lay it upon any line in your plan, which you want it ſet off upon, by ſetting the compaſſes upon it; and it will reach to the point required.

And if you want to meaſure any line upon your plan; take the length of that line in your compaſſes, and apply it to the ſcale, and it will ſhew you the length.

There are other ſorts of ſcales, as diagonal ſcales, where an inch or half an inch, is divided into 100 parts; but theſe are ſo well known, that I need ſay no more about them. Fig. 32.

If you have taken a ſurvey, and want to put it into a map; you cannot lay it down, in the bounds intended, without a proper ſcale; and it will be difficult to light of one of a proper length already made; and therefore you muſt be obliged to make one yourſelf.

PROB. II.

To project or draw the plot of a triangular field.

If the field has been meaſured with the chain; then you have all the ſides of the triangle ABC in chains, in your field book. Therefore draw any line AC for the baſe; and take the length of it with your compaſſes from the ſcale, and ſet it from A to C. Then take the length of AB from the ſcale, and ſetting one foot in A, deſcribe an arch towards B. Then take the third ſide BC from the ſcale, in your compaſſes, and ſetting one foot in C, croſs the former arch in B. Then from the point of interſection, draw the lines BA, BC; and BAC is the triangle required. 11.

If the perpendicular and baſe be known, and either ſegment AP. You muſt ſet off AP equal to that ſegment, and raiſe the perpendicular PB of the length given, and draw BA and BC.

Otherwiſe, from an angle or two.

If you have two ſides AC, CB; and the angle ACB. Draw AC, and with the compaſſes ſet off its length in chains from the ſcale. Then by the protractor, or line of chords, make the angle ACB of the given degrees; and draw CB, on which ſet off its length, (from the ſcale) to B; and draw AB.

Fig. 11. Or if the ſide AC, and the angles A and C be given. Draw AC of its due length taken off the ſcale. Then make two angles at A and C, equal to theſe given; and draw AB, CB to interſect in B.

Laſtly, if you have given AC, AD, DB, and angle ADB. Make the angle ADB equal to that given. And DA, DB of the lengths given; and AC equal the given baſe. And draw BC, BA.

Cor. *This is the foundation of deſcribing any poligon; for they may always be divided into triangles. And all the triangles may be deſcribed one after another, as above.*

PROB. III.

To lay down the plan of any field upon paper; from the meaſures taken in the field; and plotting offsets.

15. If the field has been divided into triangles, and all the ſides have been meaſured with the chain; as ABCDE; where the lengths of the ſides are ſet down in the field book, or elſe written upon the ſides, in a foul draught. Draw any line DE, and from the ſcale ſet off its length in chains, from D to E. Then with the length of EA and one foot in E, deſcribe an arch; and with the length of DA and one foot in D, croſs that arch in A; then A is another angle of the field. In like manner having the three ſides AD, DB, AB, lay down the triangle ADB, gives the angle B. And from the three ſides DC, DB, BC, lay down the triangle DBC, and we have all the ſides and angles of the figure.

16. But if all the lines are drawn from a point within as F. Then with the three ſides DE, EF, DF, lay down the triangle DEF; and in like manner the triangle EAF; then ABF; then BCF; and laſtly CDF. For in each triangle, there is given all the three ſides.

If

Fig. 20. 21. If the ſides and included angles are given, as in the figures ABD. Draw the line AS of the length given; and make the angle ASB equal to the given degrees; drawing SB, making it of its due length, and draw AB; then you have the triangle SAB. Then from the ſides SB, SC and the included angle BSC, or elſe ASC, you will project the triangle BSC, in the ſame manner; as alſo CSD; and ſo on till the whole figure is laid down.

22. When all the ſides round the figure are given, and an angle oppoſite to each of them, as in the figure ACE. Firſt draw the ſide EA from the given angle, and ſet off its length to A. Then make the angle AEB equal to that given, drawing EB. Then take AB in your compaſſes, and with one foot in A, croſs the line EB in B, and draw AB. Then make the angle AEC or BEC equal to the angle given; and with BC in your compaſſes and one foot in B, croſs the line EC in C; and draw BC. Then make the angle AED or CED equal to that given; and take the length of CD in your compaſſes, and with one foot in C, croſs the line ED in D, and draw CD; and you have laid down the whole figure. But *note*, you ought to know whether the angles ABE, EBC, ECD be acute or obtuſe.

23. When all the angles at one point or ſtation, oppoſite to the ſides of the field are given; and all the alternate angles round the field; and only one line FA meaſured. Draw the line FA of its due length; and make the angles AFB and FAB as given in your book, or foul draught; and the interſection of FB, AB, gives the point B. Again, make the angles BFC, FBC equal to their given angles, and BC, FC will interſect in C. Proceed thus round the figure, to find the points of interſection D and E. 24. But if the ſtation be taken within the field at S, and FS meaſured. Then SF, and

Fig. the two angles at S and F, will determine the point
24. A. Then SA, and the angles at S and A, will
determine the point B. And the like for the points
C, D, E. And at laſt you will have the whole
figure.

25. When all the ſides and angles, in going round it, are meaſured, as in the figure ABCDE; where *a*, *b*, *c*, *d*, *e*, are the ſeveral ſtations. Make the angle *eab* equal to that given, and make *ab* equal to its given length. Then make the angle *abc* equal to its degrees, and *bc* equal to its number of chains. In like manner make the angles *bcd*, *cde*, *dea*, of their reſpective number of degrees; and the ſides *cd*, *de*, *ea*, of their reſpective lengths in chains; and the figure is deſcribed, all but the offsets.

And to deſcribe and plan all the offsets. At *a* ſet off the diſtances to the hedges at A. Alſo ſet off, in the line *ab*, the diſtances *af*, *ag*, *ah*, *ai*; and at *f*, *g*, *h*, *i*, raiſe perpendiculars, to *ab*, of their proper lengths, all taken from the field book; and at *b* make the offset to the hedge at B. Then thro' A and B, and the ends of all theſe perpendiculars, draw the hedge AB. Then at *b* and *c*, make offsets to B and C, and ſet off *bk* to *k* where it touches the hedge; and draw the hedge B*k*C. Then make the proper offsets at *c* and *d*, to C and D; and draw the hedge CD. Again, make an offset at *d* to D, and ſet off *dl*, where it touches the hedge; and *dm*, and at *m*, the offset; and at *e* an offset to E. Then draw D*l* and *l*E thro' the end of the perpendicular at *m*, for another hedge. Laſtly, ſet off two offsets from *e* to E, and from *a* to A, and draw the hedge EA. Then you have the true figure of the field laid down.

3. Again, let AB be any ſtationary line. Set off from A, the diſtances A*c*, A*d*, A*f*, AB, from the book. And on the points *c*, *d*, *f*, B, raiſe the

perpendiculars *co*, *de*, *fg*, B*b*, of their due lengths, Fig.
taken from the book; and thro' the ends *o*, *e*, *g*, *b*, 17.
draw the hedge or boundary A*oegb*. And the like 18.
for any others.

General Obſervations.

All the lines belonging to the ſame figure muſt be taken from the ſame ſcale, with a pair of compaſſes; and it is beſt to uſe a diagonal ſcale, becauſe the links can be expreſſed by hundredth parts; and the ſcale ought to be as large as the paper will permit, that it may contain the whole draught, and the bigger it is, the more exactly you will work. All offsets belonging to each line, muſt be ſet in their proper places, and of their due lengths, perpendicular to the ſtation line; and to be taken off the ſame ſcale. And every line that is inclined to the horizon (as in a hill ſide), muſt be firſt reduced to a horizontal line, before it be laid down.

And all angles muſt be laid down with a protractor or line of chords of a good radius. And theſe things muſt be laid down in their proper poſitions, places, and magnitudes; whether they be lines, angles, or offſets, or any remarkable objects that are to be determined; and all according to the directions of your field book, or foul draught, you plan it from. And all this according to the known rules of practical Geometry. And at laſt, lines drawn thro' all the points thus determined, will give the true figure of the field.

PROB. IV.

To plan a field, as ſurvey'd by the infallible.

Let PQ be the table with the paper of obſerva- 17.
tions; where there was but one ſtation, taken in 18.
the middle of the field. Unfold the paper of ob-

ſervation,

Fig. ſervation, and upon a vacant place of the ſame pa-
17. per, take a point S for your ſtation. Then lay one
18. ſide of a parallel ruler upon the line 1, ſet the other
ſide to the point S; and by its edge, draw the line SA, always the ſame way from S; that the line 1 lies from the center of the table; and ſet from S to A its length 16.20 chains. Then ſet one ſide of the rule to the line 2, and the other ſide to S, and draw the line SB equal to 10.12 chains. Then ſet to the line 3, and to S, and draw SC 14.68. Then ſet to the line 4 and to S, and draw SD 12.90. Then ſet to the line 5 and to S, and draw SE 7.25. Then ſet to the line 6 and to S, and draw SF 8.84. Then ſet to the line 7, and to S, and draw SG 14.65. Then draw the lines ABCDEFGA for the field.

18. But if the ſurvey was taken by going round the
19. field. Let RT be the table. Unfold the paper,
and chuſe any point A for the beginning. Lay one ſide of a parallel ruler upon the line 1, and ſet the other to the point A, and draw the line AB by its edge, to lye the ſame way from A, that the line 1 lies from the center. This muſt be obſerved in drawing all the lines; and make AB 13.10 chains. Set one ſide of the rule to the line 2, and the other to B, and draw the line BC, 6.18. Set one ſide to the line 3, and the other to C, and draw CD 17.75. Set one ſide to the line 4, and the other to D, and draw DE 10.32. Set to the line 5, and to E, and draw EF 10.45. Set to the line 6 and to F, and draw FG 9.00. Set to the line 7, and to G, and draw GA, which ought to be equal to the meaſured length 8.15, if the work is right. So your field is planned.

PROB. V.

To lay down the plot of a field, as ſurveyed by the needle.

Let ACF be a field ſurveyed by the needle, going round about, taking the ſtations at *a*, *b*, *c*, *d*, *e*. 26. Thro' all the ſtations draw the meridians *am*, *bm*, *cm*, *dm*, *em*, as you come at them. Then beginning at *a*, make the angle *mab* equal to the bearing of the ſide *ab*, north-eaſterly, and thereon ſet its length in chains to *b*. The meridian *bm* being ſuppoſed to be drawn, make the angle *mbc* the bearing of *bc* ſouth-eaſterly, and make *bc* its length. Make the angle *mcd* the bearing of *d* ſouth-weſterly, and make *cd* its diſtance. Make the angle *mde* the bearing of *e*, ſouth-weſterly, and *de* its diſtance. Make the angle *mea* the bearing of *a* north-weſterly, and and *ea* its diſtance. Then the plan is finiſhed, excepting the offsets, which are laid down as directed in Prob. III. Thus, in going from *a* to *b*, you have two offsets at *s* and *t*, at B you touch the hedge. From *b* to *c*, you have four offsets at *b*, *f*, *h*, *i*, and you touch the hedge at C. From *c* to *d*, one offſet at *k*, at E touch the hedge. From *d* to *e*, five offsets at *m*, *n*, F, G, H; at *p* touch the hedge. From *e* to *a*, two offsets at H and A. The places of all theſe, muſt be marked upon the ſtationary lines; and their lengths ſet off perpendicular thereto.

PROB.

Fig.

PROB. VI.

To lay down the plan of a field, ſurveyed from two ſtations, by taking the angles round about, and meaſuring only the line between the ſtations.

27. Chuſe a convenient point upon your paper, for your firſt ſtation, as S. Thro' S draw a line, and ſet thereon the ſtationary diſtance ST; then T is your ſecond ſtation. Then with a protractor or line of chords, make the angles TSA, TSB, TSC, TSD, TSE and TSF, each of the number of degrees, as you find them in your field book; and from S draw the lines 1, 2, 3, 4, 5, 6, out at length, numbering them as in the book. In like manner, make the angles at T your ſecond ſtation, STA, STB, STC, STD, STE, STF; and from T draw the lines 1, 2, 3, 4, 5, 6, as in the book. Then obſerve where the correſpondent lines interſect; as 1 and 1 interſect at A; 2 and 2 interſect at B; 3 and 3 at C; 4 and 4 at D; 5 and 5 at E; and 6 and 6 at F. Then A, B, C, D, E, F, are the corners of the field. Therefore drawing the lines, AB, BC, CD, &c. from angle to angle, ABCDEF will be the plot of the field. Here you muſt be ſure to take the interſections of the correſpondent lines, or thoſe numbered with the ſame figures; otherwiſe you'll get a falſe plan.

28. If the ſtationary diſtance be taken in a ſide of the field, as EF, or even out of the field as *st*; the work would have been the very ſame. For the correſpondent lines 11, 22, 33, &c. that form the angles which you muſt make about F and E, or *s* and *t*, will always interſect in the points A, B, C, &c. which determine the angles of the field.

And if there had been more ſtations, proceed in the ſame manner, to lay down the angles taken at them; and to draw lines, to interſect in the points required. Fig.

PROB. VII.

To lay down the plan of any part of a country, with ſuch objects as appear; by taking obſervations at ſeveral ſtations.

This is done much the ſame way as the laſt Prob. only there are more ſtations concerned, which require to be laid down, one after another; and the diſtances to the objects meaſured. 29.

Upon a ſheet of paper, take a convenient point S for your firſt ſtation, from which draw the line ST, and ſet thereon the ſtationary diſtance ST as in your book; then T is your ſecond ſtation. With a protractor or line of chords, ſet off the angles TSK, TSL, TSA, TSB, and TSC, as in your book. And draw the lines SK, SL, SA; SB, SC; and thereon ſet the meaſured diſtances of each, as you find them in the book. Then the places of the objects K, L, A, B, C are determined.

Then in like manner at T, make the angles STD, STE, STF, STI, equal to the angles which the ſtationary line makes with TD, TE, &c. as you find them in the book; and drawing TD, TE, TF, TI, ſet thereon, the meaſured diſtances of D, E, F, I; and then their places are determined.

Next make the angle STV equal to the obſerved angle between the ſtationary lines ST and TV; and drawing TV, ſet thereon the meaſured diſtance of the ſtations T and V, as ſet down; then V is a third ſtation. Therefore at V, make the angles with TV as you obſerved them, as TVG, TVH, and perhaps TVI; and on VG, VH, ſet their meaſured

Fig. ſured diſtances, all which you have in the field book;
29. and then the places of G and H are determined. And if you have more ſtations, and more objects, you muſt proceed in the ſame manner, to lay down the angles obſerved, and to ſet off the diſtances. And thus may the plan of a ſquadron of ſhips be taken, or the map of a river, or any remarkable objects, tho' inacceſſible.

PROB. VIII.

To draw the plan of a manor or large eſtate; part of a river, or a great town.

1. Having provided ſome paper, vellum or parchment to draw it upon; conſider how large you deſign to have your map. Then collect from your field book, the dimenſions of the eſtate in chains; and from hence you may make an eſtimate, how many chains will be contained in an inch; and conſequently what ſize your ſcale muſt be of, to plot it with; and it will be beſt to uſe a diagonal ſcale.

30. 2. Then take a convenient point A upon your paper for the firſt ſtation, and from thence draw your firſt ſtationary line AB, of its due length, from the field book; then will B be your ſecond ſtation. Then having the diſtances AC and BC, you will find the place C of a third ſtation, upon the plan. And in like manner, having CD and AD, the point D will be found for a fourth ſtation. For the longeſt ſtationary lines, and the moſt general ſtations muſt be determined firſt; and then the next depending on them, and ſo on to the leaſt or moſt minute. And ſuch lines as ſerve only to find the boundaries, muſt be drawn with black lead, to be rubbed out again.

3. Then for plotting your obſervations taken upon any of theſe lines; you muſt take the ſeveral

diſtances from your diagonal ſcale, and ſet them upon these lines; or with a thin ſcale graduated the ſame way, and applied to theſe lines, prick off theſe diſtances, by the edge. Likewiſe every angle muſt be laid down truly by a protractor or line of chords. And all theſe things muſt be done in that order you took them, as directed by your field book; firſt reducing inclined lines to horizontal ones, by Prob. 9, Sect. 1. Thus in going from A to B, when we come at *a* there is a hedge; therefore ſet the diſtance A*a* from A to *a*; and at *a* lay down the angle that the hedge *ao* makes with A*a*, and ſet the length of it from *a* to *o*, for the corner of the firſt field. Then I ſet off the diſtance from A to the next hedge, and draw that hedge in like manner as the other. Then I ſet off the diſtance of the brook from A to *b*, and draw the brook, making ſuch an angle with AB as I find in my book. In this manner I proceed till all be laid down from A to B. After the ſame manner I lay down every thing from B to C, and from C to A; and likewiſe from A to D, and from D to C. Theſe general ſtations being finiſhed; the inner and leſſer ones muſt be plotted, as from F to G, from A to I, from I to F. Then from B towards H, where I project a tree, and the croſſing of the brook, and a windmill on the right; then from H to C, where I project a houſe on the left hand. Thus you muſt go thro' all the leſſer ſtations, planning every thing you find in your book, in their due places; as all objects, and all offsets to hedges, &c. till all houſes, fields, trees, and other remarkable objects, be put into your plan; proceeding from one part to another till all the parts be laid down; and if all the parts cloſe or fill up the plan, it is a ſign the work is right. And the plan is beſt done upon paper, or at laſt pricked off upon your vellum, by laying the paper upon it. And make always the north Fig. 30.

ſide

Fig. ſide of the map the uppermoſt. To do all this
30. there is required no more than a little practical Geometry, attending duly to what you have given in your field book. Sometimes it may be convenient to plot different parts in different papers; and put them altogether at laſt.

4. All hills muſt be ſhadowed, to diſtinguiſh them in the draught; likewiſe ſhadow the buttreſſes of ſteeples very deep; and alſo towers, caſtles, houſes, and all ſuch things.

5. Put the trees in each hedge, into their proper places in the map; as they have been taken on a foul draught; and make prick lines on the outſides of the hedges, for the gutters; and theſe will ſhew what hedges belong to each field. And where there are gates, the hedges muſt break off.

6. All the fields and cloſes being laid down in their true order and ſituation; write the name of each field ſomewhere in the middle of it; and if you think proper, the number of acres it contains: tho' ſome people chooſe to have their contents in a book on purpoſe.

7. Colour the hedges with different colours, and put into your map, the manor houſe, and all other out-houſes, barns, ſtables; alſo rivers, roads, bye-lanes; and any windmill, water-mill, bridge brook, pond, wood, coppice, foot-way, or horſe-way. Draw trees in woody grounds, ſhadow mountainous grounds with hills and valleys. Expreſs bogs, fens, groves, highways, rivers, gates, ſtiles, &c. Draw the meridian line, and put a mariners compaſs in ſome bye place; and a ſcale of equal parts or chains; and laſtly the title of the map, thus.

A true and perfect ſurvey of the manor
(Townſhip) of ——, in the county of ——,
Being (part of) the eſtate of ——
Anno Domini——.

8. In a very great eſtate, there ought to be drawn vertical and horizontal lines thro' the map, denoted by letters at the top and bottom and ſides; for the ready finding any field therein, mentioned in a table. Fig. 30.

9. The ſame rules, by which any field is laid down, in the method of circulation, will equally ſerve for laying down the ſtreets of a town. For take the point B for the firſt ſtation, and draw BF, and make the angle FBC equal to the degrees obſerved, as in the ſurvey book; and drawing BC, ſet its length upon it to C, taken from the book. And in that line from B to C, make offsets on each ſide, and ſet off ſuch diſtances as you have noted down. In the ſame manner, ſet off the angles BCD, CDE, DEF, and EFB. And ſet off the diſtances CD, DE, EF, FB; and make ſuch offſets on each, as you found in going along them; as the ſurvey book will direct. Then draw the inner ſtreets, till all the part BCDEF be planned. In like manner lay down the part BFGA, and ſo one part after another, till the whole is laid down. But it is beſt to lay down every part as ſoon as it is ſurveyed, whilſt you have it in your mind. 31.

PROB. IX.

To draw a map or plan of a whole county.

The laying down the plan of a county is upon the ſame principle, as laying down a ſurvey, taken from two ſtations, as delivered in the 6th Problem. Only here we are obliged to have more ſtations, and to take more angles. And as a county is of a large extent, every thing muſt be laid down with the utmoſt exactneſs. Therefore to do this, chuſe a convenient point upon your paper for your firſt ſtation. From that point draw your ſtationary line, Fig. 31. and ſet upon it, from that point, the diſtance meaſured between your two main ſtations, taken from a proper ſcale; and this gives your ſecond ſtation upon the paper. And the diſtance between them is the only meaſured diſtance you have. Then take a good inſtrument for laying down angles, and place its center at the firſt ſtation; and make an angle there with the ſtationary line, equal to the angle you took (to any certain place) at the firſt obſervation, when at the firſt ſtation; which your book will ſhew. Then make an angle with the ſtationary line, at the ſecond ſtation, equal to that you took (to the ſame place) at your ſecond ſtation; where this line (terminating the angle) cuts the other, is the place at which that particular mark ſtands: thus one place is determined. Again, at the firſt ſtation, make an angle with the ſtationary line; equal to that you obſerved, to a ſecond place. And likewiſe an angle, at the ſecond ſtation, which was obſerved there to the ſame place; and where this line interſects the firſt, is the place of the ſecond mark; ſo a ſecond place is determined. And thus go on, to lay down two correſpondent angles, taken out of the book, one at one ſtation, and the other at the other; and the interſection of the lines of that angle, gives the correſpondent place. And this work is always to be done both at the main ſtations, and at all inner ſtations.

1. All the lines ought to be taken from a diagonal ſcale, of a mile or half a mile to an inch, according to the largeneſs of the county; which may be eaſily collected from the dimenſions of the deſigned map, compared with the dimenſions of the county, in miles.

2. All your work muſt be laid down on a paper ſpread upon a flat table, and paſted down at the ends, or otherways fixt, ſo as not to be altered, till the whole be laid down. Or one may take a fine

copper plate, the bigneſs of the map, to project upon; and one ſet of obſervations may be laid down after another, as they are taken at the two ſtations; till the whole be compleated; and the lines may be drawn with a fine ſteel point. But here ſuch obſervations as are taken to one hand, muſt be laid to the contrary hand, as in all engraving. Fig.

3. And the inſtruments made uſe of in laying down the plan, muſt be exceeding good and large. Your protractor or line of chords, or ſector, or whatever inſtrument you uſe to lay down angles, muſt be of as large a radius, as the inſtrument you took the obſervations with, and as accurately divided. For moſt of your operations will be to lay down angles; and if they be not as exactly laid down in your map, as they are taken by obſervation, the map will be erroneous. And you might as well not have taken them exactly, as not lay them down exactly. Here the diſtances muſt all be reduced to horizontal lines, before they be laid down. And all the lines and angles are beſt laid down in the order they were taken; as your book of obſervations will ſhew. And the principal and moſt eminent ſtations muſt be planned firſt; and then others that depend immediately upon them; and the next, thoſe that depend upon theſe; and ſo on. And thus every thing muſt be laid down, according to what you find in your book.

4. When all the towns and remarkable places are thus determined and mapped. The hills and mountains muſt be ſhadowed, the ſea coaſts drawn; all towns, caſtles, houſes, churches exactly drawn, the largeſt of them in perſpective. All roads and rivers delineated, and all bridges drawn; and every thing properly ſhaded, to diſtinguiſh them and ſhew what they are. All the diviſions of the county muſt be drawn in prickt lines. And the name of each part written therein. And the boundaries

Fig. will be more viſible and diſtinct, if they be coloured with water colours. All this is a work of great nicety.

5. Write the names of each diviſion in great letters within it; and the names of all towns and other places, beſide them. The cities muſt be diſtinguiſhed from market towns, and theſe from leſſer towns, &c. by certain marks. Meridians and parallels muſt be drawn through the map; alſo a mariners compaſs in ſome bye place; and a ſcale of miles; the longitude and latitude of places at the ſides and ends, and the title of the map in large characters. And in all maps, take notice, that the north ſide muſt be uppermoſt.

PROB. X.

To reduce one triangle into another of equal area, to have a given baſe, or a given perpendicular.

33. Let ABC be the triangle to be reduced, and AD the new baſe. From the vertex C draw the line CD, and from B the end of the baſe, draw BE parallel to it, interſecting the oppoſite ſide in E; then draw ED, and the triangle AED is equal to the triangle ACB. For the part FBD is equal to the part EBC, being contained between the ſame parallels EB, CD; and the part AEB is common to both triangles.

And if the perpendicular be given; let it cut the ſide at the point E. Draw EB to the oppoſite angle B; and CD parallel to it, from the vertex C, to cut the baſe AB in D; draw ED, then AED is the triangle required; which is proved as before.

PROB. XI.

To reduce one triangle to another equal triangle, from a given point in the ſide.

33. Let ABC be the triangle, and E the point given. From the given point E, to the oppoſite angle B, draw the line EB. And from the vertex C, the line CD parallel to it, to cut the baſe AB in D. Draw the line ED, and the triangle AED is equal to the triangle ACB.

For the triangle EDB is equal to the triangle ECB, being contained between the ſame parallels; therefore to each of them add the triangle AEB, and then AED is equal to ACB.

SCHOL.

This Problem will be uſeful, when the area is not to run beyond a given point E.

PROB. XII.

To reduce a poligon to an equal triangle, from any angle, or point given in a ſide.

34. This is done by taking away one angle after another, till the whole be reduced. Let the poligon be ABCDEF, and let the point be at the angle D. Produce any ſide, that does not join upon D, as AF. Draw the firſt diagonal AC, and BH parallel to it, to cut the baſe FA in H. Draw HD to the next angle, and CI parallel to it, cutting AF in I. And draw DI for one ſide. Here AC, BH, HD, &c. muſt be obſcure lines.

Again, draw DF, and EG parallel to it, to cut the baſe AF in G; then draw DG for the other ſide.

Fig. 34.

In ſhort thus.

From AC open your parallel ruler to B, to cut AF at H. From HD open to C, to cut at I; and draw ID.

From DF open to E to cut at G, and draw DG. Then IDG is equal to ABCDEF.

For by reaſon of the parallels CA, BH, the triangle CBA is equal to the triangle CHA. Therefore inſtead of the two ſides CB, BA; the ſingle ſide CH may be taken. Again, by reaſon of the parallels DH, CI; the triangle DHC is equal to the triangle DHI; and therefore inſtead of the two ſides DC, CH; the ſingle ſide DI may be taken. In like manner the triangle DEF is equal to DGF; whence the triangle IDG is equal to the figure ABCDEF.

Cor. 1. *Hence any poligon may be reduced to fewer ſides, by taking away one or more ſides of it.*

Thus ABCDEF is reduced to HCDEF, and that to IDEF, and that to IDG.

Cor. 2. *Hence a triangle may be reduced back to a polygon, or a polygon to another of more ſides. Which is uſeful in reduction, when the area is to terminate at a certain point, as* F.

Thus to reduce GD to the point F. Draw from F, to the next angle D, the line FD; and from G, the parallel GE, to cut the other ſide DE in E, if there be ſuch a ſide; otherwiſe take any point E in the parallel GE, and draw the two lines DE, EF.

Likewiſe to reduce DI to the point H; draw HD to the next angle D, and IC parallel to it, in which take the point C, and draw CD, CH.

Again, to reduce CH to the point A: draw AC to the firſt angle C, and HB parallel to it, and take B therein, and draw CB, BA.

Fig. 34.

SCHOL.

It will be beſt to take ſuch a ſide for the baſe, and ſuch a point for the vertex, of the triangle (which may be in a ſide, as well as an angle), that when the operation is finiſhed, the height and baſe of the triangle may not be very unequal; obſerving that the higher the vertex is, the ſhorter the baſe will be. If the point for the vertex be taken in a ſide, it is the ſame thing, as if the poligon had one ſide more.

PROB. XIII.

To caſt up the content of a triangular field.

Let ABC be the triangle propoſed. Having got the baſe AC, and perpendicular BP, either by meaſuring them in the field; or elſe meaſuring them by the ſcale, upon the draught, laid down upon paper. Then 11.

Multiply the baſe by the perpendicular, in chains and decimals. And take half the product for the ſquare chains, which divide by 10, gives the acres, and decimal parts: or, which is the ſame thing, from half the product, cut off five figures for a decimal. If you have a mind to find the roods and perches, multiply the decimal, firſt by 4, and then by 40.

Note, you may multiply half the baſe by the perpendicular, or the baſe by half the perpendicular, for the content in chains, &c.

Fig. 11.

Examp.

Baſe AC 8.27
Perpen. BP 4.25

4135
1654
3308

Prod. — 35.1475
10) 17.5737
Content 1.757 acres.

Or thus.

When two ſides AC, CB, and the included angle C, are given; or when AC, DB, and angle ADB are given.

Add together the logarithms of AC, CB, and ſine of the angle C; or elſe of AC, DB, and S.ADB; and from the ſum abate 10. Find the number anſwering to that logarithm. Then half that number divided by 10, is the content in acres.

Examp.

AC	8.27	—	0.91750
CB	6.88	—	0.83758
S.<C	38 20	—	9.79255
	35.29	—	1.54763

Content 1.764 acres.

Or thus.

When a ſide AC, and the adjoining angles A, C, are given. To twice the log. baſe AC, add the log. ſines of the angles A and C; from the ſum, take the log. ſine of the ſum of theſe angles (or of the third angle B), and throw away 10. Find the number of this logarithm; and divide half of it by 10, for the content. Fig. 11.

Examp.

AC — 8.27	0.91750
	0.91750
S.<C 38 20	9.79255
S.<A 55 30	9.91599
	21.54354
S.<B 86 10	9.99902
35.04	1.54452

Content 1.752 acres.

Or thus, by the laſt data.

Divide the ſquare of the baſe AC, by the ſum of the natural cotangents of the angles A, C. Half the quotient divided by 10, gives the area.

8.27	1.2647		
8.27	.6873		
5789	1.9520	)68.393	(35.04
1654		58 560	1.752 content in acres.
6616		9 8330	
68.3929		9 7600	
		73000	

Otherwiſe thus.

When the three ſides are given. Find half the ſum of the three ſides, and ſubtract each ſide from it. Then multiply that half ſum and the three re-

Fig. 11. mainders continually ; and the ſquare root of the laſt product, divided by 10, gives the area.

Rather add their four logarithms together, and take half of the ſum ; and the number being found, the tenth part of it is the area in acres.

Examp.

AC	8.27,	
CB	6.88	
AB	5.14	
	20.29	Log.
½ ſum	10.14	1.00604
1 diff.	1.87	0.27184
2 diff.	3.27	0.51454
3 diff.	5.00	0.69897
		2.49139
	17.61	1.24569
Content	1.761 acres.	

PROB. XIV.

To caſt up the content of a ſquare, a rectangle, or of any parallelogram.

Multiply the length, by the breadth (which is the neareſt diſtance from one ſide to the other) ; and the product, divided by 10, is the area.

Examp. 11.

Length 16.27
Breadth 10.60

97620
16270

172.4620

Content 17.246 acres, which may be reduced to roods and perches, by multiplying by 4 and 40.

PROB. XV.

To caſt up the content of a trapezium, ABCD.

Having found the diagonal BD, and perpendiculars AF, CG, either by meaſuring in the field, or upon paper, after it is laid down. Then 12.

Multiply the diagonal, by the ſum of the perpendiculars; and half the product, divided by 10, is the content.

Examp.

Suppoſe the diagonal and perpendiculars, the ſame as were meaſured in Prob. XII. Sect. I.

1 perp. 5.18 Diag. 8.37
2 perp. 4.86 10 04

10.04 33 48
8370 0

84.0348
Content 4.202 acres.

Or thus.

13. Where the lengths of the diagonals, and the angle they make, are given. Multiply the diagonals and the nat. ſine of the angle, continually; and the product divided by 10, gives the content.

Or rather, add the logarithms of the diagonals, and the log. ſine of the angle together, rejecting 10; find the number belonging; and divide half of it by 10 (or the whole by 20), gives the content.

Examp.

Suppoſe the angle and diagonals, as meaſured in Prob. XII. Sect. I.

AC,	7.54,	0.87737
DB,	6.17,	9.79028
S.AEB,	66 20,	9.96184
	42.61,	1.62949

Content 2.130 acres.

PROB. XVI.

To caſt up the content of any field, in form of any polygon.

14. Since we do not always want to make a draught of a field, but only to find its content; in ſuch caſes, we only meaſure ſuch lines as are neceſſary for finding the content; and then fewer meaſures will do, than for plotting.

The ſhorteſt way then to find the content, is to divide the field into trapeziums, as few as you can, or into trapeziums and triangles; and to meaſure the diagonals and perpendiculars in the field, and ſet them down; and likewiſe the baſes and perpendiculars of the triangles, if there be any. Then find the contents of each of the trapeziums, by the laſt Prob. and of the triangles, by Prob. XIII. and the ſum of all is the content of the field. Fig. 14.

Examp.

Let the meaſures be taken as in Prob. XIII. Sect. I.

1. *Trapezium* CDEF.

1 perp.	3.54	Diag.	11.88
2 perp.	2.52		6.06
	6.06		7128
			71280
			71.9928

2. *Trapezium* BCFG.

1 perp.	4.47		8.19
2 perp.	3.72	Diag.	12.00
	8.19		1638
			819
			98.28

The triangle ABG.

baſe	10.56		71.9928
perp.	3.32		98.28
	2112		35.0592
	3168		205.3320
	3168	Content	10.2666 acres.
	35.0592		

Otherwiſe

Fig. 14.

Otherwiſe thus.

When we have not got the diagonals and perpendiculars, requiſite for caſting up. The field muſt be laid down on paper, by Prob. III. And then, by dividing it into trapezia and triangles, we muſt meaſure theſe as before. Therefore what we have to do, is, to meaſure the lengths of theſe diagonals, and baſes (with the ſame ſcale the field was drawn by) in chains and decimal parts; and their correſpondent perpendiculars, by taking the neareſt diſtance from the vertex or angle, to the baſe or diagonal.

Then multiply the diagonal of each trapezium by the ſum of its perpendiculars; and the baſe of every triangle by its perpendicular; and take half the ſum of all theſe products, which divide by 10, gives the acres and decimals. And the decimal may be reduced to roods and perches, if you pleaſe, by multiplying by 4 and then by 40.

And *note*, if any part of the area caſt up is without the field, ſuch part muſt be ſubtracted from the whole.

As for the offsets, and parts of irregular curves, if there be any; they muſt be computed ſeparately, and added to the former contents, if they lie without the ſtationary lines; or ſubtracted, if within. To find their contents will be ſhewn in the following Problem.

Otherwiſe thus.

4. Reduce the field into a triangle by Prob. XII. And if any ſide in it be very irregular, running in and out, a right line may be drawn by the eye, that takes in, as much as it leaves out. Then the baſe of this triangle multiplied by its height, and half the product divided by 10, will give the area in acres, as uſual.

This reducing a field to a triangle, is as exact a way as caſting up the contents of all the trapeziums and triangles ſeverally; and far more eaſily and ſpeedily done. Fig. 34.

Examp.

Let ABCDEF *be any field; which is reduced to the triangle* IDG. *And let the baſe* IG *be* 9.00, *and the perp. or the neareſt diſtance from* D *to* AF 6.46.

6.46
9.00

58.14

Content 2.907 acres in the triangle.

Therefore the content of the polygon is 2.907 acres.

SCHOL.

In uneven hilly ground it has been diſputed, whether the horizontal baſe, or the true ſuperficies, ought to be computed; and the greateſt part of ſurveyors ſeem to be agreed, that the ſuperficies ſhould be meaſured. And indeed, ſince ſurveying is but a mechanical practice, we may lay down this general rule, that ſince we meaſure by the chain, we muſt compute by ſuch dimenſions as we find by the chain. But if we compute from a map or draught, we muſt needs take our meaſures from the map, to compute by. And in theſe caſes there will always be a difference between the area meaſured and the area planned; more or leſs, as the ground is more or leſs hilly and irregular; and ſubject to aſcents and deſcents, and to hills and dales. But tho' the ſuperficies ought to be meaſured, only the horizontal baſe muſt be planned; for otherwiſe a map cannot be made too cloſe, *i. e.* for the adjoining parts to come together.

Fig. 34. It may be obſerved, that ſince all ground is more or leſs irregular, as well as the fences that incloſe it; therefore if ſeveral perſons ſurvey the ſame piece of ground; no two of them will give exactly the ſame content. Nay, if even the ſame perſon ſurveys it twice, he will differ from himſelf more or leſs; eſpecially if he ſurveys it by different methods.

If any geometrical curve is given to be ſurveyed; you muſt meaſure it by the rules of menſuration, for that purpoſe; for I cannot here lay down rules for all ſorts of figures, but ſuch as are common, and ſuch as come in practice every day. But for geometrical curves of all ſorts, I muſt refer to that book. But theſe things rarely come in queſtion.

PROB. XVII.

To caſt up the contents of offsets and parts of irregular curves.

1. The parts of a field contained by offsets are trapezoids and triangles; and therefore they muſt be meaſured as ſuch. And to find their contents, multiply each part of the ſtationary line by the ſum of the offsets on each end of it, and this being repeated as often as there are offsets; add them all together, and take half the ſum, and divide it by 10, gives the whole area contained by theſe offsets; which muſt be added or ſubtracted, according as they lay within or without the field.

If the offsets are at equal diſtances, find a mean offset, by dividing the ſum of them all, by their number. Then multiply this mean offset by the whole baſe, for the content in chains, which divide by 10.

Examp.

Fig. 3.

Examp.

Suppoſe AB *be the ſtationary line, and* Aoegh *the hedge; and let the offsets be as in* Prob. III. Sect. I. *To find the area* AB*hgeo*A.

Offset at A 0.
at *c*, 0.87
0.87
A*c* 1.94
3 48
783
87
1.6878

Offset at *c*, 0.87
at *d*, 0.23
1.10
cd, 1.48
880
440
110
1.6280

Offset at *d*, 0,23
at *f*, 0,81
1.04
df, .75
520
728
.7800

Offset at *f*, 0.81
at B, 0.47
1.28
*f*B, .28
10 24
25 6
.35 84
1.68 78
1.62 80
.78 00
4.4542

Content .2227 in acres,
for the area of AB*hgeo*.

And after the ſame manner muſt all the offset areas be found in the field ADF, Fig. 26, and ſubtracted from the whole content, as they lay without the field.

2. If

Fig. 2. If any part of the area be bounded by a regular curve, and the height be ſmall, multiply the
3. baſe by the height, and take $\frac{2}{3}$ the product for the content in chains.

35. 3. When ſome part of the ground is bounded by an irregular curve, as *abc*, meaſure the baſe AC. Then for finding the content of AC*ca*; erect the perpendicular ordinates or offsets A*a*, B*b*, C*c*, at equal diſtances, and meaſure them. Then

$$\frac{Aa + Cc + 4Bb}{6} \times AC,$$ will be the area of AC*ca*, in chains.

Or more exactly, if *ad* be a curve; divide the baſe AD into 3 equal parts at B, C, and meaſure it, and the perpendicular ordinates A*a*, B*b*, C*c*, D*d*; then $\frac{Aa + Dd + 3Bb + 3Cc}{8} \times AD$, will be the area of AD*da*, in chains.

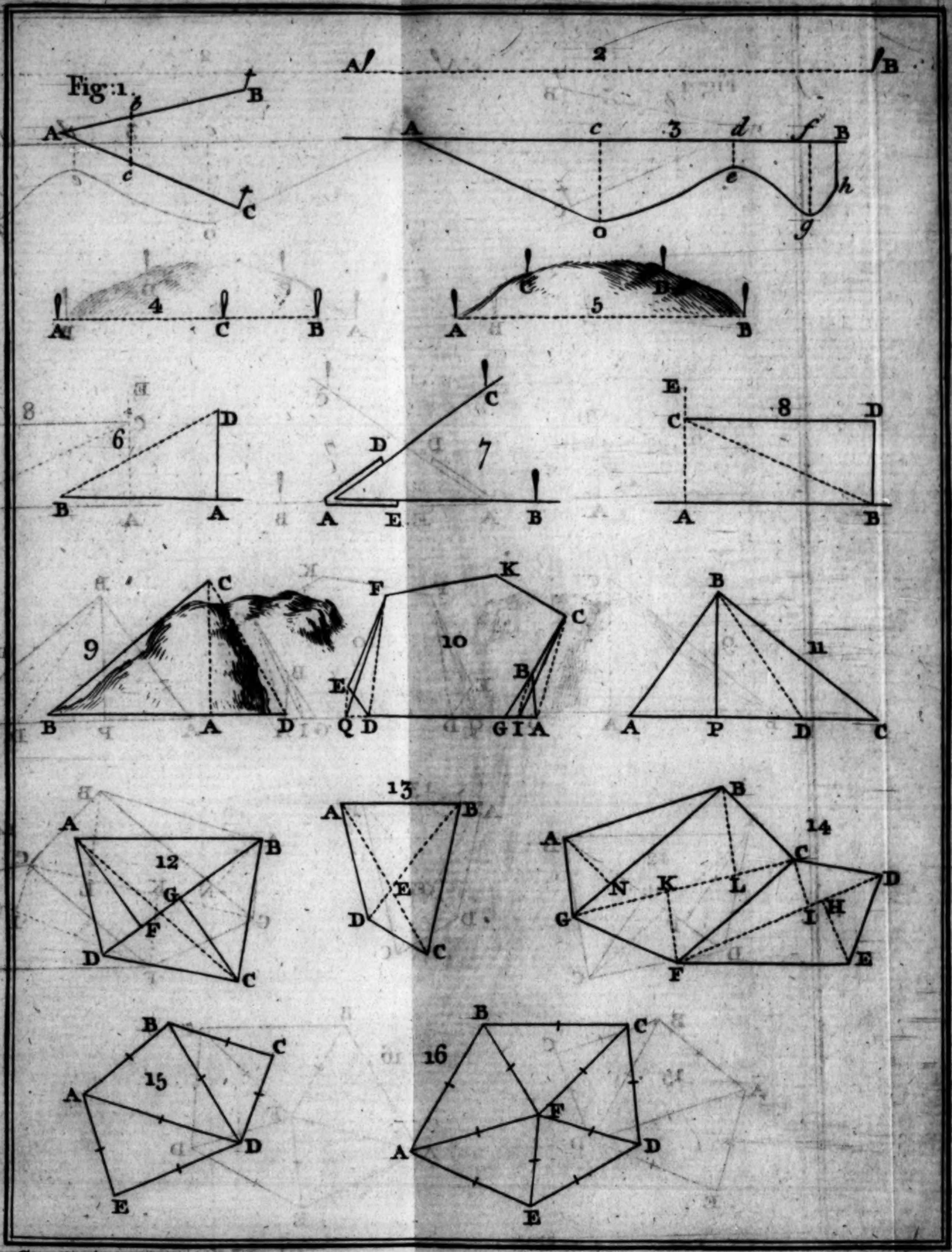
Or more exactly ſtill. Divide the baſe AE into four equal parts, at B, C, D; and meaſure the offsets or ordinates, A*a*, B*b*, C*c*, D*d*, E*e*, (as in Cor. Prob. III. Sect. I.) then will

$$\frac{7 \times \overline{Aa + Ee} + 32 \times \overline{Bb + Dd} + 12Cc}{90} \times AE$$ be the area of AE*ea*, in ſquare chains.

SCHOL.

If a lordſhip is to be ſurveyed; after all the fields have been meaſured and laid down in the plan, the contents of them all are to be caſt up, one by one, by ſome or other of the rules and directions beforegoing; and their contents written in the field, or elſe put into a table, to have recourſe to upon occaſion. And ſuch a table or book is beſt made, to have the names of the cloſes go on alphabetically; eſpecially if the lordſhip be large, and contains a great number of cloſes.

PROB.

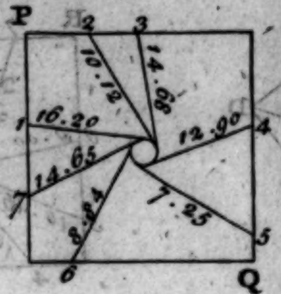


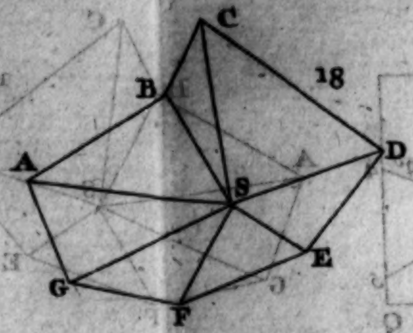
Surveying

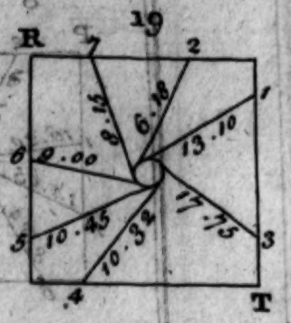
Pl:I. pa:90.

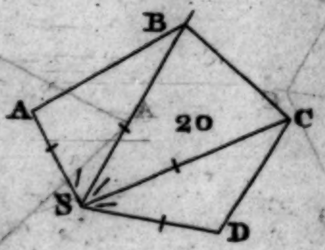
GR III
BRITISH MUSEUM

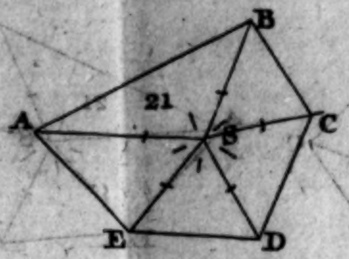
Fig:17.

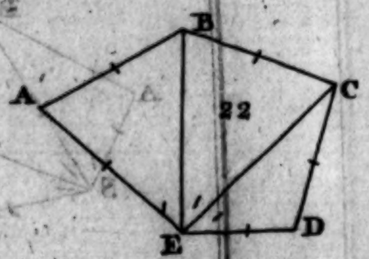


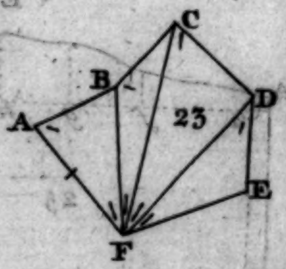


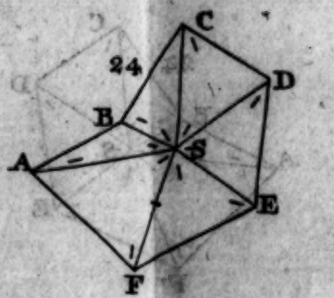


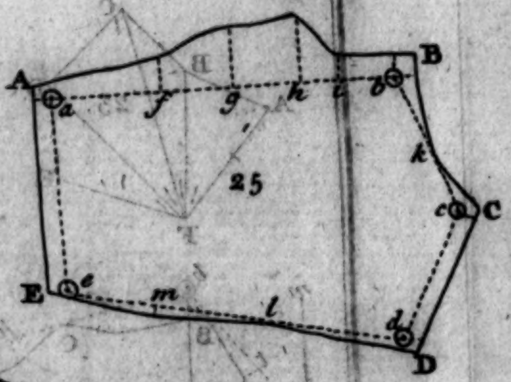


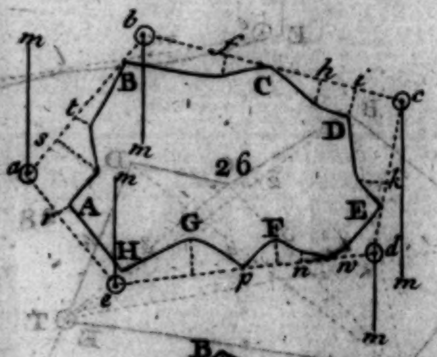


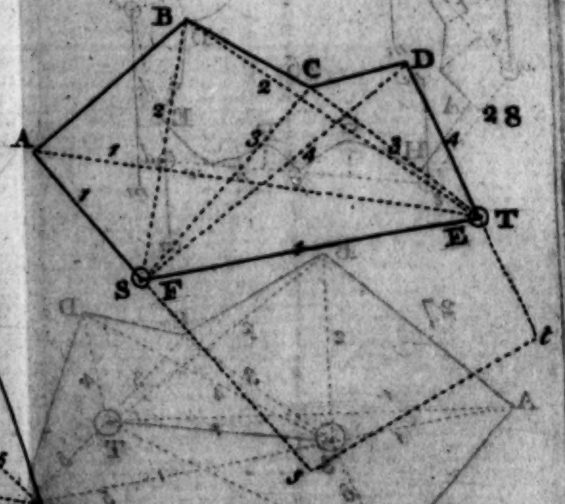


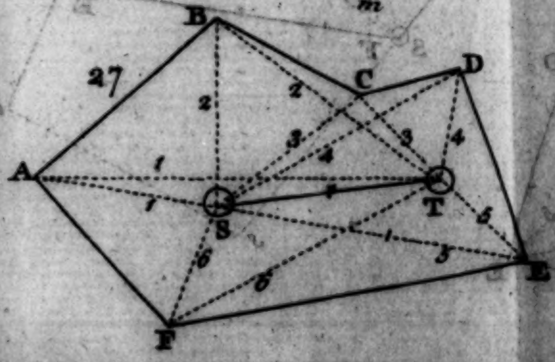












Surveying

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BRITISH MUSEUM

PROB. XVIII.

To lay out any number of acres in a triangle, whoſe baſe and containing angle is given.

Multiply the baſe AC by the nat. ſine of the angle C, for a diviſor, by which divide 20 times the acres given; and the quotient is the ſide BC. 36.

Or thus by logarithms. To the log. of the number of acres, add the conſtant log. 11.30103, and reſerve the ſum. To the log. baſe AC, add the log. ſine of the angle C; and ſubtract this ſum from the former ſum. The number of the remaining log. is the ſide CB.

For if A be the number of acres, then $\frac{AC \times CB \times S.C}{2 \times 10} = \text{area} = A$, therefore $CB = \frac{20A}{AC \times S.C}$.

Examp.

Suppoſe 17.64 *acres to be laid out in a triangle, whoſe baſe* AC *is* 24.81 *chains; and angle* C, 38 20. *To find the other ſide* CB.

17.64,	1.24649	
conſt. log.	11.30103	
	12.54752	
AC 24.81,	1.39463	
S.C 38 20,	9.79255	
	11.18718	
CB 29.92,	1.36034	chains.

Cor. *If you would lay out any number of acres in a triangle, of a given height.*

Fig. Divide twenty times the number of acres, by
36. that height or perpendicular; the quotient is the length of the baſe, for any triangle.

PROB. XIX.

To lay out any number of acres in a rectangle, whoſe baſe is given.

Divide ten times the given number of acres, by the baſe given; and the quotient is the height of the rectangled parallelogram.

Examp.

Suppoſe 64.25 *acres laid down in a rectangle, one ſide of which is* 18.27 *chains; to find the other ſide.*

```
18.27)642.5(35.16 the height.
      5481
      ----
       9440
       9135
       ----
        3050
        1827
        ----
        1223
        ----
```

Cor. *If the acres are to lie in a ſquare; extract the ſquare root of ten times the number of acres given, for the ſide of the ſquare.*

Fig.

PROB. XX.

37. *To divide a triangle in any ratio, from an angle; or to cut off any part of it, by a line drawn from an angle* B.

Divide the oppoſite ſide AC in the point G, in the ſame ratio that the triangle is to be divided; and draw the line BG.

Or if the area be known; ſay as the whole area ABC, to the part to be cut off ABG; ſo is the whole baſe AC, to the part AG to be cut off, then draw the line BG.

As if you want $\frac{1}{2}$, $\frac{1}{3}$ or $\frac{1}{4}$ of the triangle cut off, make AG to be $\frac{1}{2}$, $\frac{1}{3}$ or $\frac{1}{4}$ of AC, and draw BG.

For triangles on the ſame baſe, and of the ſame height, are as the baſes; therefore ABG : ABC :: AG : AC.

Cor. 1. *If you want to cut off a given quantity from the triangle, without knowing its content; proceed by* Prob. XVIII.

Cor. 2. *If you would divide the triangle into any number of equal parts; it is only dividing the ſide* AC *into the ſame number of equal parts; and drawing lines to theſe points, from the oppoſite angle.*

PROB. XXI.

To divide a triangle ABC *in any proportion, by a line* DE, *drawn parallel to a given ſide* BC: *or to cut off any part of it.*

Divide either ſide AC, in the given proportion, at F. And make AE a mean proportional between AF and AC. Thro' E draw ED parallel to BC, which divides the triangle as required. 38.

Fig. 38.

Or thus.

Say as the whole triangle, to the part next the angle A to be cut off; ſo is the ſquare of either ſide as AB, to a fourth number; extract the ſquare root of it, which ſet from A to D, and draw DE parallel to AC, for the dividing line.

For the triangles ABC and ADE are ſimilar; therefore ABC : ADE : : $AB^2 : AD^2 : : AC^2 : AE^2$: : AC : AF.

PROB. XXII.

To divide a triangle ABC *in a given ratio, by a line drawn from a given point* D *in a ſide; or to cut off any part of it.*

39. From the given point D, draw a line DB, to the oppoſite angle B. Then divide AC in F in the given ratio; and from F draw FG parallel to DB, to cut CB in G; then draw DG, which divides the triangle as required; the part DGC being at FC; and the part ADGB, as AF.

For drawing BF, the triangles GFB, GFD, contained between the ſame parallels, are equal; add to both CFG, then CFB is equal to CDG; and therefore either of them is to the whole CBA, as CF to CA.

Cor. 1. *If you want to divide the triangle* ABC *into any number of equal parts; divide the baſe* AC *into the ſame number of equal parts; from all which points draw parallels to* BD; *and where they interſect the ſides* AB, CB, *draw lines from thence to* D.

Cor. 2. *If you want to cut off a given quantity from the triangle, without knowing its content; proceed according to* Prob. XVIII.

PROB.

Fig. 40.

PROB. XXIII.

To divide a triangle ABC *in a given ratio, or cut off any part of it, by a line* GI *perpendicular to any ſide* AC.

Draw BD perpendicular to the ſame ſide AC. And let r repreſent the part to be cut off, and s the whole area. Then take $AG = \sqrt{\frac{r}{s} \times AD \times AC}$, and at G erect the perpendicular GI, then AGI is the part required. If G falls between D and C, take the other part $(s - r)$, and meaſure from C.

For, triangle AGI : triangle ACB : : $AG \times GI : AC \times BD$: : (by ſimilar triangles) $AG^2 : AC \times AD :: \frac{r}{s} \times AD \times AC : AC \times AD :: r : s$.

PROB. XXIV.

To divide a triangle ABC *into three equal parts,* 41. *by three lines drawn from the three angles,* A, B, C.

Make AD a third part of AC; thro' D, draw DE parallel to the next ſide AB. Biſect DE in F; and from F, draw to the three angles A, B, C, the three lines FA, FB and FC, which will divide the triangle as propoſed.

For ſuppoſing a line drawn from B to D, the triangle ABD would be $\frac{1}{3}$ of ABC, but ABD and ABF are equal, being contained between the ſame parallels; therefore ABF is $\frac{1}{3}$ the triangle ABC. Therefore the two equal triangles AFC and BFC are equal to $\frac{2}{3}$ of ABC, and each is $\frac{1}{3}$ of it.

Cor. *Hence a triangle* ABC *may be divided into* 3 *parts, by* 3 *lines drawn from the three angles, which ſhall be as any three numbers* a, b *and* c.

Fig. For make AD to AC as *a* to *a* + *b* + *c*; and 41. draw DE parallel to AB. Then make DF to DE as *b* to *b* + *c*. And draw AF, BF and CF. Then it is plain, AFB is as the number *a*, and AFC as *b*, and BFC as *c*. For AFD : BFE : : DF (*b*) : FE (*c*) : : DFC : EFC. Or AFC : BFC : : *b* : *c*.

PROB. XXV.

To divide a triangle ABC *into 3 parts from a given point* D *within it; to be to one another, as the numbers a, b, and c.*

42. In any ſide AC, take AH to AC, as *a* to *a* + *b* + *c*, and draw DH, and from the oppoſite angle B, draw BF parallel to DH; and draw DB and DF, then ABDF is the part repreſented by *a*.

In like manner take CG to AC, as another of the numbers *b*, to *a* + *b* + *c*; and draw DG, and BI parallel to it; and draw DI. But as I falls without the figure, to reduce it (by Cor. 2. Prob. 12.) draw DC and IN parallel to it, to cut BC in N, and draw DN; then BDN is the part repreſented by *b*. Therefore FDNC is the third part repreſented by *c*.

For ſuppoſing BH drawn, then by conſtruction ABH will be equal to *a* = ABF + FBH = ABF + FBD = ABDF.

Again, if BG be ſuppoſed drawn; then by conſtruction GBC = *b* = GBI — CBI = BDI — BCI = (DOI being a right line) BDO — OCI = BDN, becauſe DON = COI, by reaſon of the parallels DC and NI.

Cor. *By the ſame way, one may divide a triangle into as many parts as he pleaſes, from a given point* D *within it; and in any ratio aſſigned.*

But ſuch Problems as theſe, are more for the exerciſe of Geometry, than for any great uſe they are of in ſurveying. I ſhall only add another, about triangles, of the ſame ſort. Fig. 42.

PROB XXVI.

To divide a triangle ABC *in a given ratio, by a line* IH, *drawn thro' a given point* D.

Make AF to AC as the part AHI to the whole ABC, and draw BF, and alſo DG parallel to AC. 43. Then put $P = \frac{AB \times AF}{DG}$. And take $AH = \frac{1}{2}P + \sqrt{\frac{1}{4}P - AG \times P}$, and draw IDH. And when AH comes out greater than AB, work for the angle C, inſtead of A.

For, the triangles AHI and AFB will be equal, whence $AH \times AI = AF \times AB$, and $AI = \frac{AF \times AB}{AH}$. And by the ſimilar triangles HGD, HAI, $HG : GD :: HA : AI$ or $\frac{AF \times AB}{AH} :: AH^2 : AF \times AB$. Whence $HA^2 \times GD = HG \times AF \times AB = \overline{HA - AG} \times AF \times AB = HA \times AF \times AB - AG \times AF \times AB$, and $HA^2 = \frac{AB \times AF}{GD} \times HA - \frac{AB \times AF}{GD} \times AG = P \times HA - P \times AG$. Which reduced, and the root extracted, gives $HA = \frac{1}{2}P + \sqrt{\frac{1}{4}PP - AG \times P}$.

Cor. *If the point given, is without the field, at* d; *then will* $AH = \frac{1}{2}P + \sqrt{\frac{1}{4}P + Ag \times P}$.

For then *g* being on the other ſide of A, A*g* will be affirmative.

Fig. 44.

PROB. XXVII.

To divide a parallelogram in any ratio, or to cut off any part of it; by a line parallel to one of the ſides.

Divide the other ſide AB at E, in the given ratio, and make CF equal to BE, and draw the line EF, which will divide the parallelogram as was required.

If you want a certain part of it cut off; take BE the ſame part of BA, and CF of CD; and draw EF.

If you deſire to cut off ſo many acres, without knowing the content of the whole; divide your number of acres by the breadth of the parallelogram, EG; and the quotient will ſhew how far you are to meaſure along the ſide BA or CD; where you muſt draw a line, parallel to the ſide BC or AD, as EF.

Cor. *Hence, you may divide a parallelogram into any number of equal parts; by dividing the oppoſite ſides,* AB *and* DC *into the ſame number of equal parts, and drawing lines thro' the correſpondent points.*

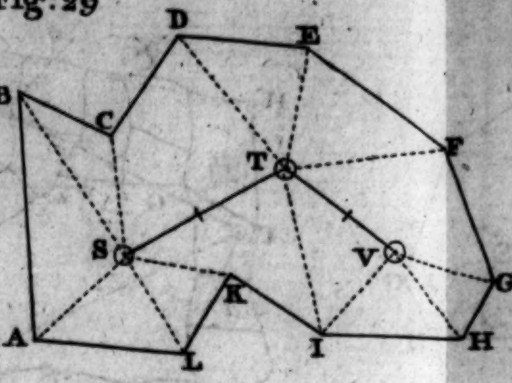
PROB. XXVIII.

To divide a trapezoid in a given ratio, or to cut any part of it; by lines croſſing the parallel ſides.

45. Divide the parallel ſides AB, DC in the ſame ratio, and to the ſame hand, as at E and F; and draw the line EF, which will divide the trapezoid in the ſame given ratio.

Otherwiſe, to cut any part off it; make AE the ſame part of AB, and DF the ſame part of DC; and draw EF.

Fig: 29



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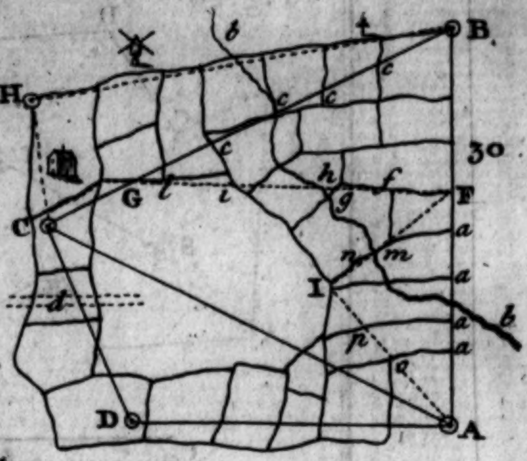

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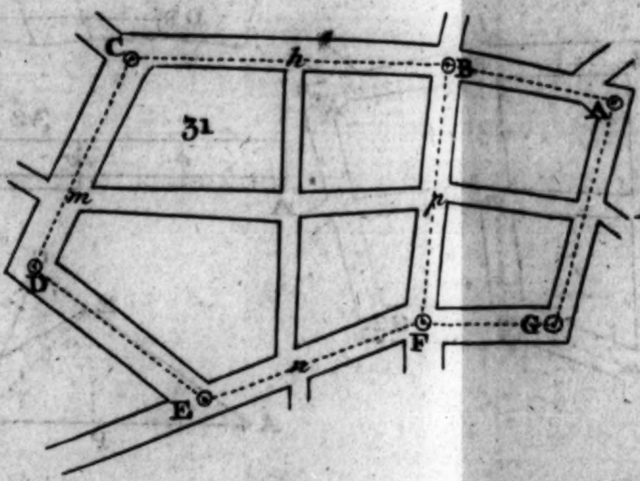

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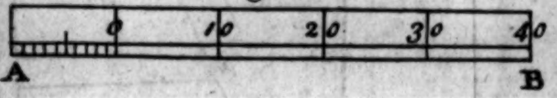

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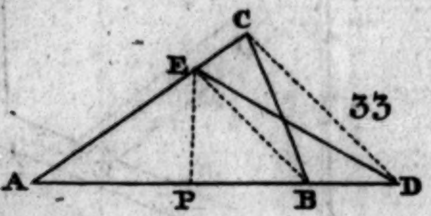

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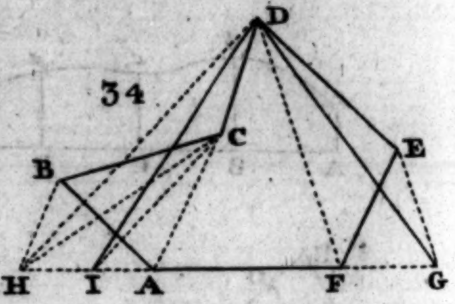

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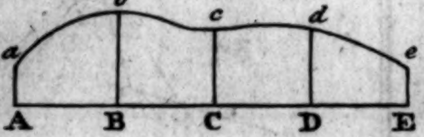

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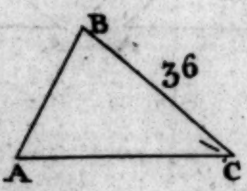

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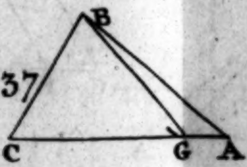

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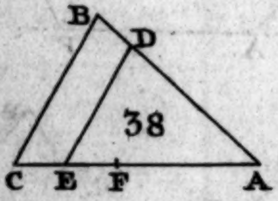

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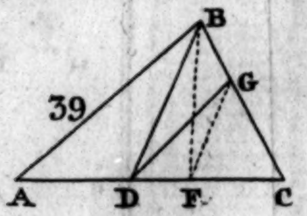

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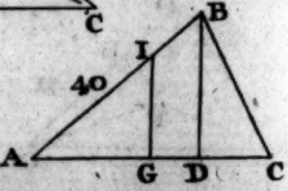

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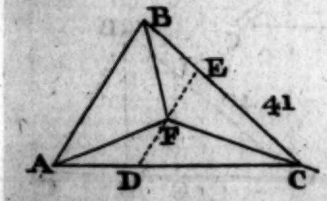

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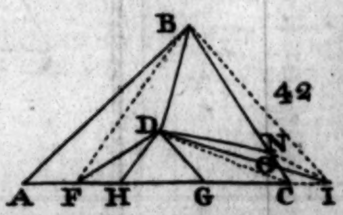


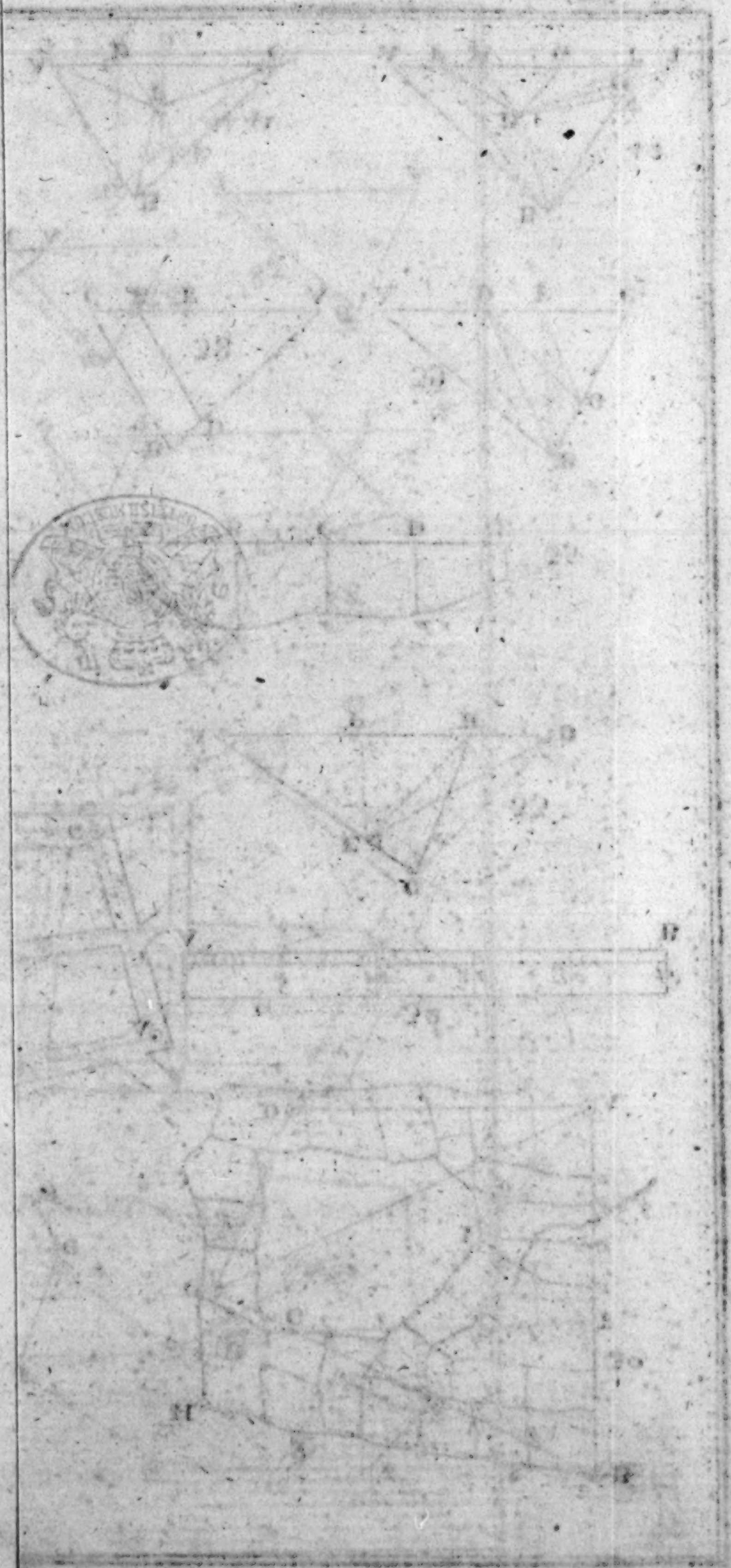


Fig. 45. Cor. *Hence a trapezoid may be divided into as many equal parts as we pleaſe; by dividing the two parallel ſides, into the ſame number of equal parts; and drawing the correſpondent lines.*

PROB. XXIX.

To divide a trapezium in any given ratio of the parts, from an angle A.

46. Draw the diagonal BD oppoſite to the given angle A; and divide it at I, in the ratio given; and draw the lines AI and IC, which will divide the trapezium as required.

But, if you would have one right line drawn from A to divide it; draw the other diagonal AC, and IF parallel to it, to cut DC in F, then draw AF, which will divide it in the given ratio.

For the triangle AID is to AIB, in the ratio of DI to IB, that is in the given ratio; and CID is to CIB in the ſame ratio; therefore the part ADFI is to the part ABCI in the given ratio. But by reaſon of the parallels AC, IF, the triangle AIC is equal to AFC; and therefore ABCI is equal to ABCF; and ADCI equal to ADF.

PROB. XXX.

To divide the trapezium ABCD, *in any given ratio, by a line drawn from* E, *a point in the ſide* AB.

47. Produce the oppoſite ſide DF; and reduce the trapezium ABCD into the triangle GEH, from the point E, by Prob XII. Then divide the baſe GH at F, in the given ratio, and draw EF, which will divide the trapezium as required.

If the point F falls not between D and C, but between C and H; then BC muſt be produced for

the

Fig. 47. the baſe, inſtead of DC. Or elſe reduce it back, by Cor. 2. Prob. XII.

For the part DAE is equal to DGE, and ECH equal to ECB. Therefore EADF is equal to EGF, and EBCF equal to EHF, whence theſe parts are as GF and FH, that is, in the given ratio.

PROB. XXXI.

To cut off any quantity of land from a trapezium, by a line parallel to one ſide, as CB.

48. Divide the number of acres, given to be cut off, by the ſide CB, to which you are to go parallel. And the quotient ſet out from B to E, and from C to F; and draw EF. But if EF is greater than BC you muſt ſcarce go ſo far, from B or C; or if it be leſs you may go ſomething further. But when this is ſet out, as the field is irregular, you muſt meaſure the quantity cut off CBEF, and ſee what it is over or ſhort. Then meaſure FE and reduce that difference of the areas, to ſquare, chains, which divide by the length of FE, and the quotient muſt be ſet from E towards A, if you had cut off too little, as to *e*; and from F to *f*. But the contrary way if you had cut off too much. Then draw *ef* for the true line of diviſion.

Note, you may ſet it all to one end if you like; and then you muſt double the quotient, and ſet it from E to *n*, on the greater ſide, and draw *n*F. And indeed the courſe of the ridges in a field often confine us; ſo that when we expect they run parallel to the hedge, we find them otherwiſe; and therefore we may lay out more at one end than the other, in theſe caſes.

Cor. *Hence we may apply this method to any poligon; provided the ſides* AB, CD, *that we work upon, be not far from parallel. Otherwiſe a proper allowance*

ance muſt be made for the increaſe or decreaſe of the line EF. Fig. 48.

PROB. XXXII.

Any poligon, regular or irregular, being given; to divide it in a given ratio, from a point in an angle or a ſide; or to cut off any quantity from it.

Let ABCDE be the poligon; reduce it into a triangle CFG, by Prob. XII. whoſe vertex is C, and baſe FG. Then divide FG in the point I, in the ratio given; and draw the line CI, which divides the triangle FCG, and alſo the poligon ABCDE, as was required. 49.

Note, if CI do not cut AE, but ſome other ſide AB or ED, then that ſide it cuts muſt be produced for the baſe inſtead of AE, the point C being ſtill the vertex; or elſe it may be reduced by Cor. 2. Prop. XII.

If a given quantity of acres was to be cut off; reduce that quantity to chains, and take the neareſt diſtance from C to AE for the perpendicular; divide twice that number of chains by this perpendicular, and the quotient will be the length of the baſe, which ſet from F to I, then the line CI drawn, will cut off the quantity propoſed ABCI.

Otherwiſe thus.

If the ratio of the two parts be given, and the whole quantity; then either part is eaſily found.

Therefore to cut off any quantity towards CB, meaſure ſo many triangles next that ſide, as CBA, CAE, &c. till you have ſomething too much (or wants a ſmall matter); then cut off the overplus from the laſt triangle (or the defect from the next); and draw the line of diviſion.

The method to cut off the due part of the laſt triangle is this, let CAE be the triangle, then divide

Fig. 49. vide twice the exceſs (reduced to chains) by the perpendicular from C to AE; and the quotient ſet from the laſt angle E to I, gives the point of diviſion.

Examp.

The field ABCDE contains 42 acres, the part CBAI to be cut off is 27 acres. The triangle CBA is 10.80; the triangle CAE is 24.46. The perpendicular 22.7.

10.80
24.46

35.26
27.00

over 8.26 = 82.6
chains

82.6
2

22.7)165.2(7.28 = EI.
1589

630
454

1760

Therefore if EI be made 7. 28. chains, and CI drawn; the part CBAI will contain 27 acres.

Cor. *By the ſame method, a poligon may be divided into any number of equal parts; and even into any number of parts, whoſe proportions are expreſſed by any numbers.*

This is done either by dividing the baſe of the triangle FG in the ratio of the ſeveral parts. Or elſe cutting off the ſeveral parts of the figure one after another. And when there are many parts, it Fig. comes to no more, than dividing a triangle into ſeveral parts, by lines from the vertex, as explained 49. in Prob. XX.

This problem is of very great uſe in practice; and by theſe methods any pieces of land may be divided more expeditiouſly, and as truly as by any method whatſoever. And for meaſuring any poligon, when it is reduced to a triangle, it is done at once; which otherwiſe would require as many calculations, as there are triangles.

PROB. XXXIII.

To divide a poligon into any number of parts, equal or unequal; by lines drawn from a point within it.

To do this, you muſt have the content of every triangle in the field, and of the whole field. And then beginning at any angle, draw a line to that angle, and take ſo many of the firſt triangles as will make the firſt part required; and what they fall ſhort of that part, cut off the next triangle. Then take the remainder of that triangle, and ſo many of the next, as will make up the ſecond part. And ſo proceed round the figure, till all the parts be laid out.

Examp.

Let ABCDEF *be a field containing* 69 *acres, which* 50. *is to be divided among three men, from a pond in the middle at* G; *ſo that every man may have the benefit of the water. And the firſt man to have* 25 *acres, the ſecond* 20, *and the third* 24. *To ſet out every man's ſhare from the angle* C.

Draw the line GC for the beginning of the diviſion; and alſo lines to all the reſt of the angles, as GB, GA, GF, GE, and GD. Then meaſure all the

Fig. the triangles; and their contents will be as fol-
50. lows.

CGB	11.81
BGA	15.27
AGF	6.07
FGE	6.65
EGD	16.83
DGC	12.37
	69.00

Then ſince the firſt man's ſhare is 25 acres, the firſt triangle CGB 11.81 is too little, and the firſt and ſecond triangles together, CGB and BGA, comes to 27.08, which is more than 25; therefore ſubtract 25 therefrom, and there remains 2.08. And therefore we muſt cut off 2.08 acres from the triangle BGA towards A. To do which, meaſure the perpendicular from G to BA, 13.28 chains, and reduce 2.08 acres to chains, which will be 20.8; divide 41.6 or double theſe chains, by 13.28; and you have for the quotient 3.13 chains; which ſet off by meaſure, from A towards B, as to H, for the dividing point. And draw GH, ſo CGHB is the firſt man's ſhare.

Then for the next man's ſhare. Add this 2.08 acres cut off, to the next triangle AGF 6.07, the ſum is 8.15 which is too little, add the next triangle FGE 6.65, makes 14.80 acres, which is ſtill too little; and adding the next triangle EGD, it will be far too much. Therefore we ſhall cut off an additional part of the triangle EGD at E. Subtract 14.80 from 20 the ſecond man's ſhare, and there remains 5.2 acres, or 52 chains. The perpendicular from G upon ED is 13.9 chains. Therefore divide 104 (the double of 52) by 13.9, and the quotient is 7.48 chains; and this muſt be meaſured

ſured off from E towards D, as to I. Then draw GI, and HGIEFA will be the ſecond man's ſhare. Fig. 50.

Laſtly, the remainder of the triangle GDE, is 11.63 equal to the part IGD; to this add the triangle DGC 12.37; and the ſum is 24 acres for the area GIDC, which is the third man's ſhare. So the field is divided as was required.

If the field was to be divided into any number of equal parts; divide the whole content of the field by the number of parts, the quotient ſhews the quantity of one part or ſhare. As if there were 12 equal ſhares in it, then one ſhare will be 7.5 acres. Therefore cut from the triangle CGB, 7.5 acres; and from that and the next 7.5 acres more, and ſo on; which may be done as before directed, or by Cor. Prob. XVIII.

If you deſire to cut off any number of acres given, without knowing the content of the whole field. You muſt meaſure triangle after triangle, till you get as much as you want, or very near it. And for the defect cut off a part of the next.

PROB. XXXIV.

To transfer a plan or map truly, from one paper to another.

Lay your plan upon the blank paper, and fix them both together upon ſome flat board or table, that they may not move from one another or part aſunder, till the operation is done. Then with a fine needle or pricker, make ſmall pricks thro' all the principal angles, or places on your plan, which will make ſo many marks in your paper. Then take off the plan, and draw lines from one point or mark to another, as you find them in the plan; till all the lines be laid down. If you be in any doubt about ſome point or prick; takes its diſtance from one

Fig. one or two points, you are certain of; and apply it to the plan, and that will ſet all to rights.

Or thus.

Take the map or plan, and rub the backſide all over with black lead; then lay your paper upon a flat table, and upon that lay the leaded ſide of the plan. Faſten them at the four corners, that they may keep together. Then with a ſharp pointed bodkin run over all the lines in your plan, laying pretty hard on; and ſuch other objects as you would expreſs. Then take off the map, and all the lines, that you traced with the bodkin, will appear upon the paper; which you may draw with ink, or may rub out at pleaſure, with crums of bread.

Otherwiſe thus.

Take a paper well oiled with linſeed oil; and laying the map upon a table, lay the oiled paper upon it, and faſten them together; then you may ſee all the lines and furniture of the map, thro' the oiled paper, as thro' a clear glaſs. Then with pen and ink, or with a bodkin, draw or trace all the lines in the map. So will you have a map in the oiled paper, which you may place upon another paper, if you pleaſe. Thus by an oiled paper, may the figure of any flower, fruit, or other curious thing, be taken from any draught of it.

Or thus.

The drawings in a map may be transferred to another paper, by taking every line from the one, and ſetting it upon the other, with a pair of compaſſes: but this is very tedious, and requires a deal of time; but to facilitate the work, there are three legged compaſſes made, to take off three points at once, and ſet them down on the new paper.

PROB.

Fig.

PROB. XXXV. 51.

To reduce any field to a greater or leſſer ſize, in any proportion of the ſides.

Take a point in a ſide or an angle, as at A. From that point, draw lines to all the other angles, as AE, AD, AC. Then on any ſide or diagonal, ſet off the length of that ſide or diagonal, according as you would have it reduced; ſuppoſe along AF, from A to *f*. From *f* draw *fe* parallel to FE, to interſect the next diagonal AE in *e*. From *e* draw *ed* parallel to ED, to interſect AD in *d*. From *d* draw *dc* parallel to DC to interſect AC in *c*. From *c* draw *cb* parallel to CB. Then *afedcba*, wil be the reduced figure.

Or you may take a point within the figure, as at P; from which draw lines to all the angles, PA, PB, PC, PD, PE. Then make P*a* the reduced length of any line as PA; and from *a* draw *ab* parallel to AB, to interſect the next line PB in *b*. In like manner draw *bc*, *cd*, *de*, *ea*, parallel to BC, CD, DE, EA; each from the interſection of the correſpondent line drawn from P. And *abcde* will be the figure reduced, as was propoſed. And by the ſame rules, they may be reduced to a bigger ſize, as well as a leſſer. 52.

Cor. 1. *Hence the reduced figure is ſimilar to the original figure.* 52.

For the angles are all equal; and the ſides paralled, and proportional, to thoſe in the given figure.

Cor. 2. *Hence, a figure* ACE *may be divided in a given ratio, by lines parallel to the ſides of the figure; by the ſame method.*

Making Af^2 to AF^2, or Pa^2 to PA^2, in the given ratio.

Fig. 52.

PROB. XXXVI.

To reduce a map or plan, to a leſs or greater ſize, in any proportion of the ſides.

If the draught or plan is not large, lay it down again by a ſmaller ſcale; to do which you muſt take every line from the plan, and apply it to the ſcale it was laid down by; and then take the ſame numbers, off the leſſer ſcale, to be laid down in the new plan. Thus you muſt perform the whole work geometrically, by ſcale and compaſs; and it is beſt to uſe a diagonal ſcale, by reaſon of their minute diviſions. But when the plan is large, this method is tedious.

Otherwiſe.

Make a reducing ſcale with a ſmall hole, in a line with the edge of it, and nearer one end than the other, in proportion of the ſize it is to be reduced to. Theſe are both graduated with equal parts, in the ſame proportion. Then glue or fix your map to a table, and cloſe by it edge by edge a paper, on which it is to be reduced into a leſſer or greater form. Put a ſtrong needle thro' the hole in the center of the reducing ſcale, and fix it in the table about the middle of the line where their edges join. Then, if you would reduce it to a leſſer ſize; turn the edge of the longer end of the ſcale, to any object on the plan; and ſee what number of equal parts it cuts; and find the ſame number on the other end of the ſcale, and by its edge make a mark on the paper, for the place of that object. And thus you muſt proceed till all things on the map be laid down.

But if you would make a larger draught, you muſt turn the ſhorter end of the ſcale to the objects on the map, and the longer end to the paper.

The rule muſt be a thin plate, that the numbers Fig. may be pricked off by the edge of it.

But plots or maps are beſt reduced by an inſtrument contrived on purpoſe. This is made with two drawing pens; ſo that tracing one of them over all the lines in the plan, the other pen will deſcribe it anew on a clean paper, either in a leſſer or a greater ſize, as one pleaſes.

Or thus.

Incloſe the map in a large ſquare, and divide that 53. into ſeveral ſmall ſquares. Alſo make a ſquare upon 54. your new plot, ſo much leſs than the other, as you would have it diminiſhed; and divide it alſo into the ſame number of ſmall ſquares. Then obſerve in what ſquare, and what part of the ſquare, any object, on the map, is placed; then place it in the like ſquare, and in the ſame part of it, on your new plan. And thus transfer every remarkable point in the map, into your new plot; putting it in its proper ſquare, and into the ſame ſituation. Thus in fig. 53. you have ſeveral cloſes to the weſt, and a hill near them; likewiſe a river running thro' the ground, two trees near it, and a houſe a little below; with a road leading to it. And in the reduced plan, fig. 54. you have alſo the cloſes to the weſt, and hill near them; and the river running thro' the ground, and two trees beſide it, and a houſe below, and the road going to it; and all placed in the correſponding ſquares, and the ſame parts thereof. And thus all other things therein, ought to be placed. And the ſame way, a plan is reduced from a leſſer to a greater ſize; but this ſeldom comes in practice.

This method is very proper for a large plan, becauſe it contains a great variety of things.

PROB. XXXVII.

To reduce one ſort of meaſure into another.

In ſome places, thro' cuſtom, they reckon more feet to the perch than the ſtatute allows; and therefore an acre with them will be greater than a ſtatute acre. But to know how many ſtatute acres there are in any field, ſurveyed by a different perch; and the (contrary); by having that perch, and the correſpondent number of acres given. Multiply the number of acres by the ſquare of the length of the perch it was ſurveyed and planned by; and divide the product by the ſquare of the other perches length; and the quotient gives the number of acres, according to that perch.

Examp.

Suppoſe a field of 40 *acres was planned by a perch of* 18 *feet; how many ſtatute acres does it contain?*

18	16½	272,2|5)12960)47.60
18	16½	10890
144	96	20700
18	16	19054
	16¼	
324		16460
40	272¼	16332
12960		1280

Anſ. 47.6 acres.

Cor. *The length of the perch, a field is meaſured by; is reciprocally as the ſquare root of the number of acres, it meaſures to, by that perch.*

PROB. XXXVIII.

Having given the content of a field; to find what ſcale it was laid down by; or how many parts to an inch.

Take any number of parts to an inch for your ſcale, by which meaſure the content of the field; then multiply the content given by the ſquare of your ſcale (or number of parts) and divide the product by the content found by that ſcale; the ſquare root of the quotient, is the true ſcale, or number of parts in an inch.

Examp.

There is a field contains twelve acres, but meaſured by a ſcale of 10 *parts in an inch, its content is* 16½ *acres. To find the ſcale.*

```
  10    24
  10   100
  ——   ———
 100
 ———
16.5)2400(145.5,     145.5(12.07
     165.             1
     ———             ——
      750         22)45
      660             44
      ———             ——
       90             1.50
       82
      ———
        8
      ———
```

So the ſcale contains 12.07 parts in an inch.

Cor. *The ſcale or number of parts in an inch, is as the ſquare root of the content found by it, in a given field.*

S E C T. III.

Of Meaſuring inacceſſible Heights, and Diſtances.

P R O B. I.

To meaſure the height of any perpendicular object AB, *where you can come to the baſe* A.

55. LET ABD be the object, as ſuppoſe a tower, whoſe height AB is required; AC a horizontal line; for here C is ſuppoſed to be level with the foot of the object at A.

Take your ſtation at C, about as far from the object as the height of it; and where you can ſee both the top and foot of the object; and there place your inſtrument, with which take the angle BCA; that done, meaſure the diſtance CA, from the inſtrument to the foot of the tower. Then by plain trigonometry ſay,

As radius :
to the diſtance CA : :
So the tangent of the angle BCA :
to the height of the tower AB.

To the height thus found, add the height of the inſtrument above the ground.

Examp.

Fig. 55.

Examp.

Suppoſe the angle BCA 51 52, and diſtance CA, 64 feet.

Rad	—	10
CA 64	—	1.80618
Tan. 51 52	—	10.10510
AB 81½	—	1.91128

To which add 4 feet the height of the eye, the ſum is 85½ feet.

This may alſo be reſolved by projection; draw CA, on which ſet 64 feet from any ſcale, to A. Erect the perpendicular AB. Then make the angle ACB 51 52; and draw CB interſecting the perpendicular in B, the top of the tower. Then AB meaſured on the ſame ſcale, gives 81½.

Otherwiſe, by the ſhadow.

Upon plain ground, ſet up the ſtaff *ab* perpendicular to the horizon; ſo that the ſun ſhining, it may caſt the ſhadow *ac*, and let AC be the ſhadow of the tower AB. Then meaſure the ſhadows *ac* and AC, and the length of the ſtaff *ab*. Then ſay,

As the length of the ſtaff's ſhadow ac :
is to the height of the ſtaff ab : :
So is the length of the object's ſhadow AC :
to the height of the object AB.

Or thus by the ſhadow without the ſtaff. Take, with an inſtrument, the altitude of the ſun, when the ſhadow of the top of the tower B, falls at C. Then meaſure CA, and you have the angle C, and the baſe CA; to find the perpendicular BA; which is done by the firſt method.

Fig.

SCHOLIUM.

55. In taking the angle ACB, if you move backwards or forwards, till that angle be juſt 45 degrees; then the diſtance CA, being meaſured, will be equal to the height AB.

If you be on the top of the tower, you may take the angle ABC, to any mark at C; then ſubtracting the angle ABC from 90, gives the angle ACB; then meaſuring AC; from theſe, AB will be found as before. And this comes to the ſame thing as meaſuring a depth.

Hence alſo if the height AB be given, the diſtance of any point C, as AC, may be found; having the angle ACB. It is only varying the proportion, given in the firſt method.

We may obſerve, that the altitude AB, may be found otherwiſe than by calculation, if we can get to the top of it. And that is, by help of a line and plummet, let down from the top B, to the bottom at A. For the length of the line, reaching from B to A, being meaſured, gives the height BA.

PROB. II.

To meaſure an inacceſſible height DB.

56. Suppoſe DB to be a tower and a ſpire, where we cannot come at the bottom of it; or in caſe of a mote or other hindrance, between us and the object; that we can get no nearer than A; we muſt proceed thus.

Suppoſe the line of ſtation BAC, drawn from the foot of the perpendicular DB, to be horizontal. In this line we muſt take two ſtations A and C, at a good diſtance from one another. Then place the inſtrument at A, and take the angle DAB, ſuppoſe 58°; then go to the other ſtation C, and take the

angle DCB, 38 deg. Then meaſure the diſtance Fig.
between the ſtations CA, 26 yards. 56.

Then to project it, draw a line CB, and at one end A, make the angle DCB 38 deg. and draw CD. Then ſet off the ſtationary diſtance 26 from C to A; and at A make the angle BAD 58 deg. and draw AD, to interſect CD in D. Then the diſtance of D from the line CB, applied to the ſcale, gives the height 40 yards.

Or thus, by Trigonometry.

Subtract BCD 38°, from BAD 58°; there remains 20°, for the angle ADC. Then,

As	Rad —	10.
	× ſ. ADC, 20	9.53405
To	CA, 26	1.41497
So is	S.C, 38	9.78934
	× S.A, 58	9.92842
		21.13273
To the height BD, 39.7,		1.59868

To this height, add the height of the inſtrument above the ground for the whole height.

For by trigonometry, $S.{<}D : CA :: S.{<}C : AD = \frac{S.C \times CA}{S.D}$ and rad : AD or $\frac{S.C \times CA}{S.D} ::$ S.A : BD ; that is,

$$\text{Rad} \times S.ADC : CA \times S.ACD :: S.BAD : \text{height } BD.$$

$$\text{or Rad} \times S.ADC : S.ACD \times S.BAD : CA : BD.$$

Or thus, by nat. tangents.

Find the difference of the cotangents of the angles BAC, and BAD. Then,

As the dif. cotangents :
Radius : :
So the ſtationary diſtance CA :
to the height DB.

Cot.

Fig. Cot. 38 1.27994
56. Cot. 58 .62487

diff. .65507

then .655 : 1 : : 26 : DB.

1

.655)26.00(39.7 = DB.
1965

6350
5895

455

For if DB be radius, CA will be the difference of the tangents of ADB and CDB, which are the complements of A and C. And therefore rad : that difference : : DB : CA.

Otherwiſe thus.

If you take two ſtations at A and C, ſo that the angle DAB ſhall be double to DCB. Then CA will be equal to AD.

Then *Radius* :
the ſtationary diſtance AC : :
S.angle DAB :
to the height DB.

Or thus.

Take the ſtation at A, ſo that the angle DAB may be juſt 45 degrees; and the angle DCB 26° 34′. Then the height DB will be equal to the ſtationary diſtance CA.

For when DAB is 45, DB = BA = CA, and CB = 2DB. But it is, as CB (or 2DB) : DB, or as 2 : 1 : : rad : tan. 26° 34′.

Cor.

Fig. 56. Cor. 1. *Hence one may find the height of one object,* DEF, *ſtanding upon another* GEF.

Firſt find the whole altitude DB, and then the height of the tower EG, by this Prob. And their difference, is the height of the ſpire EDF alone.

Cor. 2. *If the height of the tower* EG *be known; any diſtance* CA, *in the line* GC, *may be meaſured from* E.

For that is only the reverſe of this Prob. and therefore CA may be found by working the proportion backwards, by the natural tangents; or by the ſines; as laid down before.

PROB. III.

To meaſure an inacceſſible height BD, *by means of any two ſtations; ſo that one of them* A, *may be level with the baſe* B *of the object.*

57. Draw the horizontal line BA, and taking one ſtation at A in that line, take another any where as at C, no matter whether it be higher or lower. Place the inſtrument at A, and take the angles BAD and DAC, and go to C, meaſuring the ſtationary line AC; and at C plant the inſtrument, and take the angle ACD. Then ſubtract the ſum of the angles DAC and DCA from 180, to get the angle ADC.

Then ſay, $$\text{as}\left\{\begin{array}{l}\textit{Radius}\\ \times\ \text{S.ADC}\end{array}\right.$$
$$\text{to}\left\{\begin{array}{l}\text{S.ACD}\\ \times\ \text{S.DAB}\end{array}\right.$$

So the ſtationary diſtance AC :

to the height BD.

Examp.

There is a tree DE *ſtanding upon a high hill* BE; *to find the height of its top* D, *above the level of* A.

Firſt

Fig. 57. First I place the inſtrument at A, and take the angle BAD 58 deg. and the angle DAC 72° 10′. And then I meaſure the ſtationary diſtance AC 26 yards, and at C I fix my inſtrument, and take the angle ACD 64° 30′. Therefore, ADC is 43 20. Whence,

Rad.	10.
S.ADC, 43 20,	9.83647
S.ACD, 64 30,	9.95548
S.DAB, 58 0,	9.92842
AC, 26	1.41497
	21.29887
height BD, 29	1.46240

Then to this height, add the height of the inſtrument. The demonſtration is the ſame here, as in the laſt Prob.

Or thus.

Take the ſtation A, ſo that the angle BAD may be 30 deg. Then will DA be equal to 2DB; and it will be,

As S.ADC,
to $\frac{1}{2}$CA;
So S.ACD,
to the height DB.

Otherwiſe thus.

This ſuppoſes both ſtations A and C to be level with the baſe B of the object.

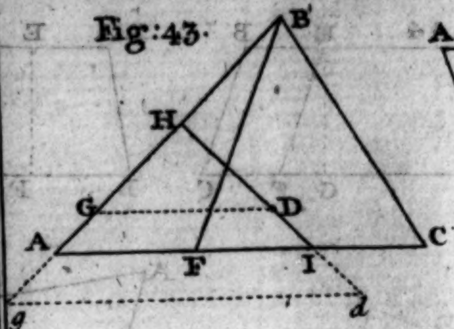
Here are given the angles of altitude BAD, BCD, and the horizontal angle at one of the ſtations, as ACB, and the ſtationary diſtance CA.

Make as tan. BCD : tan. BAD : : S.ACB : S.BAC, obſerve whether acute or obtuſe.

Then ſubtract the ſum of the angles BCA and BAC from 180, gives the angle ABC. Then ſay,

As

Fig: 43



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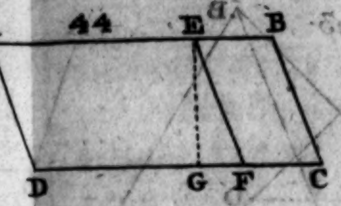

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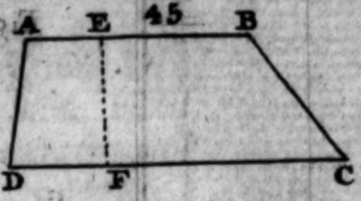

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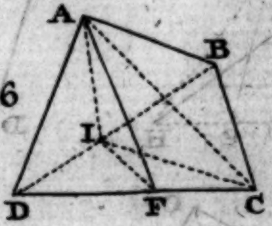

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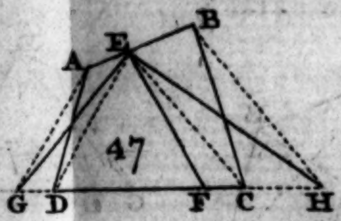

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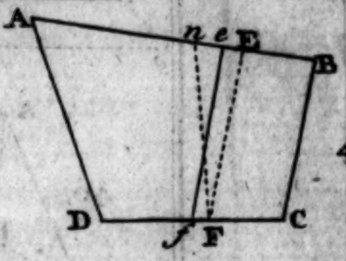

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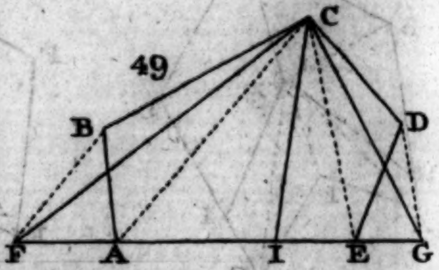

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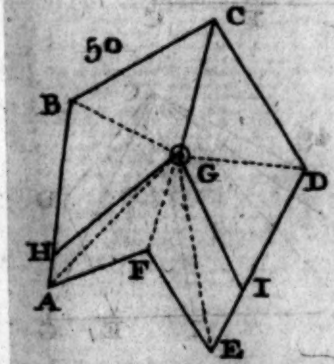

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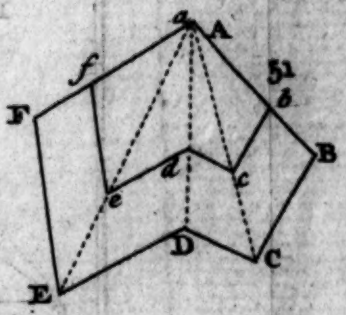

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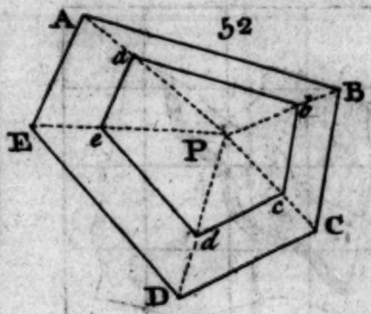

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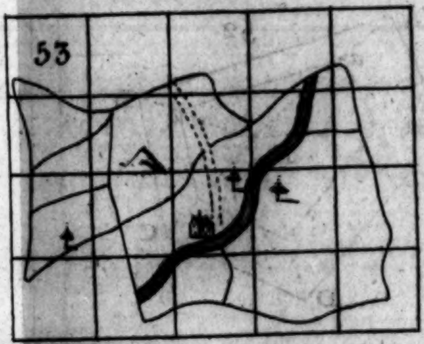

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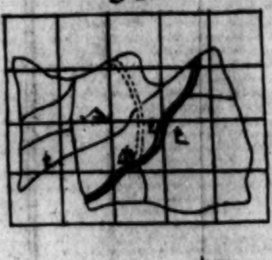

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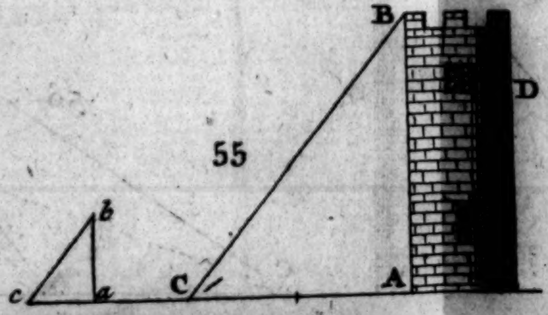

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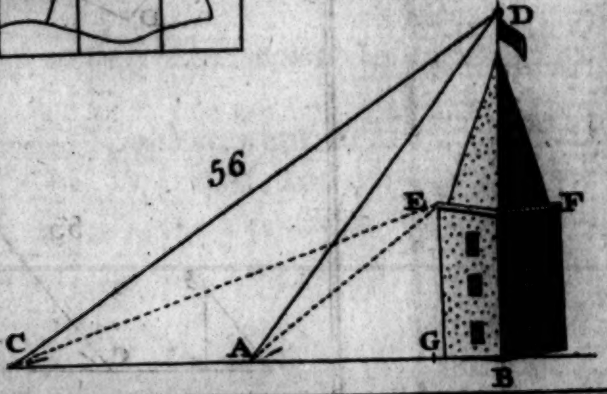

Surveying

Pl: IV. pa: 118.

As S.ABC × radius : S.ACB × tan. BCD : : ſo Fig. 57.
the ſtationary diſtance AC : to the height DB.

This method is deduced from Prob. 125 of my Algebra.

In the former method, the point C may be either in or out of the plane DAB, as is evident. For DAB is perp. to the horizon; but ADC may be any way inclined to it. But in the latter method both ADB, and CDB are perp. to the horizon; and therefore the plane DCA muſt always be inclined to it.

Cor. 1. *A depth may be found the ſame way as an altitude; by aſſuming two ſtations, from both which, any ſpot in it may be ſeen, where you can ſet up a mark.*

This is eaſily underſtood, by only conceiving the point D to be below B.

Cor. 2. *If* CA *coincided with* AB, *this would come to the ſame caſe as the laſt Prob. And if* CA *was perpendicular to the horizon, this Prob. would be the ſame as finding one altitude* BD, *from having another altitude* AC *given; which therefore is eaſily reſolved.*

After this manner the height of a hill or a mountain, or any object above the level of the place, is eaſily found.

Cor. 3. *Hence may be found the height* DE *of one object upon another; by firſt finding the height* DB, *and then* EB; *and the difference will be the height* DE.

PROB. IV.

To find the height of a very high mountain.

This might be reſolved by what went before, if 58.
it was not for the refraction of the rays of light, which do not come ſtreight to the eye from ſuch a height in the atmoſphere, where the air muſt be greatly

Fig. 58. greatly rarified. For this reaſon, a ray coming from the top of a mountain to the earth, deſcribes a curve line, concave to the earth. Therefore an obſerver at the earth, taking the angle of altitude of an object upon the top of it, directs his eye along the tangent to that curve; for in that direction the rays come to the eye. But as that tangent runs above the mountain, he makes the angle of altitude higher than it ſhould be; and of conſequence the mountain itſelf higher than it really is. Now to rectify this, let marks be ſet up at the ſtations A and C; and one at the top of the mountain at D. And let the obſervers go, one to A, and the other to D; and let them both at the ſame time, make their obſervations thus; let the obſerver at A take the altitude of D above the horizon; where the rays, inſtead of coming in the right line DA, will come in the direction FA, the tangent of the curve at A: and therefore he really takes the altitude of the point F. Alſo let the obſerver at D take the depreſſion of A below the horizon; where the rays, inſtead of coming to him in the line AD, will come in the direction FD, the tangent of the curve at D. And therefore he has taken the depreſſion of the point F, inſtead of D. Now by comparing notes, the angle at A will be greater than that at D, whereas they ought to be equal. Therefore take half the difference of the obſerved angles, and ſubtract it from the altitude taken at A, and that gives the true altitude. For the altitude at A was taken too great by the quantity DAF, and that at D too little by ADF equal to the former. The ſame thing is to be done at the ſecond ſtation C. Theſe angles being rectified, you may then proceed by either method laid down in the laſt Prob. (eſpecially the latter) to find the altitude; and for this purpoſe make your ſtationary diſtance AC as long as you well can.

It may happen that two ſtations may be taken Fig.
upon a mountain, more conveniently than below; 58.
and then you will only want one ſtation below. But this creates no material difference in the work; for it is only finding a depth inſtead of a height.

PROB. V.

To find the height of a cloud by its ſhadow.

Obſerve ſome ſmall cloud which is coming directly towards you, or going directly from you; as C. 59.
Then obſerve when the middle of its ſhadow is at ſome remarkable point upon the ground, as at A; at that inſtant take the altitude ABC of the middle of the cloud. That done, take the ſun's altitude at the place of your ſtation B, and that will be equal to the angle BAC; alſo meaſure the diſtance BA between your ſtation and the ſhadow. Then you have a triangle ABC, where all the angles and the ſide BA are given; which is eaſily laid down by rule and compaſs, and the height of C, above BA, found. Or by trigonometry. Subtract the ſum of the angles at B and A from 180, to get the angle C. Then ſay, as rad × S.ACB : S.BAC × S.ABC : : ſo the ſtationary diſtance AB : to the height of the cloud.

For $S.C : AB : : S.B : AC = \frac{S.B}{S.C} \times AB$. And rad : AC or $\frac{S.B}{S.C} \times AB : : S.A$: perpendicular; or rad × S.C : AB : : S.A × S.B : height.

If the cloud be directly over your head at the time of obſervation, then CBA will be a right angle. Then rad : AB : : tan. ſun's altitude CAB : height CB.

You muſt have a ſmall cloud to obſerve by, becauſe the obſervation muſt be made to a point. But

Fig. But if you would obſerve a large cloud, take the
59. altitude of the very edge, and obſerve the very edge of the ſhadow. And you muſt be upon ſome large plain or open ground, where you can ſee about you.

PROB. VI.

To find the altitude of an object whoſe top appears juſt in the horizon; having the diſtance in miles given.

This is a matter which frequently happens at ſea, in ſailing to or from the land.

Square the diſtance in miles, which multiply by the decimal .2223, and the product is the height of the object in yards.

60. For let AD be an arch of the earth's ſurface, DB the height of an object, AB a tangent at B. Then in the right angled triangle CAB, CB^2 or $CD^2 + 2CD \times DB + DB^2 = CA^2 + AB^2$. But becauſe $CA = CD$, $2CD \times DB + DB^2 = AB^2$, or (becauſe BD is very ſmall) $2CD \times DB = AB^2 = AD^2$, and $DB = \frac{AD^2}{2CD}$. But 2CD, or the earth's diameter, is 7916 miles; therefore $\frac{AD^2}{7916} = DB$ in miles, and $\frac{AD^2 \times 1760}{7916} = DB$ in yards; that is, $.2223 \times AD^2 =$ the height DB in yards. And the point B juſt appears or diſappears, to a ſpectator in the tangent at A.

PROB. VII.

To meaſure the length of a horizontal line AB *acceſſible at both ends,* A *and* B.

61. A line may be acceſſible at both ends, and yet cannot be meaſured, by reaſon of ſome impediment which happens to be in the way, as wood,

water, houſes, &c. In ſuch a caſe, ſet up a ſtake Fig. 61.
at ſome diſtance from the line, as at F; and thro' F produce the lines AF, BF, beyond F, in which take FD equal to one of the lines as FA, FC, equal to the other FB; and leave two marks at D and C. Then meaſure the length of the line CD, which will be equal to that required AB.

If there is not room to meaſure out the lengths AF, BF; take half of them and meaſure from F to D and C; then meaſure CD and double it. Or take $\frac{1}{3}$ of each, and then CD is $\frac{1}{3}$ of AB.

Or thus.

Take a ſtation at ſome place where you can ſee A and B, as at F; and with an inſtrument take the angle AFB, then meaſure the two lines AF, BF. And then the triangle AFB is eaſily laid down with ſcale and compaſs.

Or take the angles at A and B, or any two angles; and meaſure one ſide; and then the triangle may be laid down, or computed by plain trigonometry.

PROB. VIII.

To meaſure a diſtance AB, *acceſſible only at one end* B.

Let the end B be one ſtation, and chuſe another 62.
at a convenient place as C; and having an object at A, produce the lines AB and AC to D and E, of any length. Then meaſure the ſides of the triangle BCD, and alſo the ſides of the triangle BCE. And by theſe the diſtance BA may be determined; all this with the chain only.

Upon a paper draw the line BC, of its due length, and on that conſtruct the triangle BCD with the ſides you meaſured. Alſo upon the ſame line BC, make the triangle BCE, with the ſides, as you meaſured

Fig. ſured them. Then produce DB and EC till they
62. interſect in A; then BA meaſured, on the ſame ſcale, will ſhew the diſtance of A from B, or the length of the line BA.

It may be alſo computed by plain trigonometry. In the triangle CBD there are given all the ſides; to find the angle DBC, and its ſupplement ABC. And in the triangle BCE, all the ſides are given, to find the angle BCE, and the angle BCA its ſupplement. And laſtly, in the triangle ABC all the angles will be known, and the ſide BC, whence AB will be found.

Or thus.

Take one ſtation at B, and another at a convenient diſtance as C; and ſet up marks. Then placing the inſtrument at B, take the angle ABC, between ſome mark or object at A, and the mark at C. Then meaſure the ſtationary line BC; and place the inſtrument at C, and take the angle ACB, between the ſame object at A and the mark at B. Then ſubtract the ſum of theſe angles B and C from 180, to get the angle A. Then ſay,

As ſine of the angle A :
to the ſtationary diſtance BC : :
So ſine of the angle C :
to the diſtance AB.

The diſtance BA may alſo be found here by rule and compaſs, by laying down the triangle ABC.

Or thus.

Make the angle ABC of 60 degrees, or if you cannot at B, take ſome other point in the line BD. Then chooſe ſome other point C, in the line BC, that the angle BCA may alſo be 60. Then meaſure BC, and that is the length of BA. For if ABC be 60°, and BCA 60, then CAB will be 60; and the triangle ABC will be equilateral.

Or elſe make the angle ABC a right angle, and ſomewhere in the line BC, take the point C, that the angle BCA may be 45 degrees. Then meaſure with the chain the length of BC, and that is the length of BA. For the angle at A is alſo 45 degrees; and therefore the ſides oppoſite to the equal angles, AB and BC, will be equal. Fig. 62.

Or you may make the angle ABC a right angle, and the angle ACB 60 degrees. Then meaſure the length of BC; then BA will be equal to the ſquare root of $3BC^2$, or equal to $BC\sqrt{3}$. For in that caſe $AC = 2BC$, and $AB^2 = AC^2 - BC^2 = 4BC^2 - BC^2 = 3BC^2$.

Or make the angle ABC what you will, and the angle BCA half the ſupplement of it; then meaſuring the length of BC, you will have the length of BA. For then the angle BAC will alſo be half the ſupplement of ABC, ſince they are both together the whole ſupplement. Therefore their oppoſite ſides are equal.

PROB. IX.

To meaſure an inacceſſible diſtance AB; *or one that cannot be come to, at any end.*

Set up a ſtake ſomewhere in the line BA produced, as at C; then chuſe another ſtation as at D. Then with an inſtrument at C, take the angle ACD. Then meaſure the diſtance CD; and placing the inſtrument at D, take the angles CDA, ADB. Subtract the ſum of the angles ACD and ADC from 180, and there remains the angle CAD, or rather add the two angles ACD and ADC together, and you have the angle DAB. Likewiſe ſubtract the ſum of the angles DAB and ADB from 180, and you have the angle ABD. Then make, as S.BAD × S.ABD : S.ACD × S.ADB :: ſo is the ſtationary diſtance CD : to the diſtance ſought AB. 63.

For

Fig. 63. For $S.A : CD :: S.C : AD = \frac{S.C}{S.A} \times CD.$

And $S.B : S.ADB :: AD$ or $\frac{S.C}{S.A} \times CD : AB.$

Or $S.A \times S.B : S.C \times S.ADB :: CD : AB.$

It may alſo be laid down by projection. Draw the line CB, and make the angle BCD equal to that obſerved; ſet off CD, the diſtance meaſured; and at D, make the obſerved angles CDA, ADB; and the part AB intercepted between DA and DB, is the diſtance required.

Or thus.

64. When you cannot get a ſtation in the right line AB; take a proper ſtation at F, where you can ſee A and B; and produce the lines AF and BF, to D and C, and make FD and FC, of any convenient lengths, by meaſuring them. Then placing the inſtrument at F, take the angle AFB. Then go to C and take the angle ACB; and likewiſe at D, the angle ADB. Then the work may be laid down thus.

Draw any line AD, and at any point as F make the angle AFB equal to that obſerved, and draw the line BFC; and ſet off the diſtances FD and FC. Then at the point C, make the angle BCA, equal to that you meaſured, drawing CA to interſect DA in A, one end of the line. Again, make the angle ADB equal to its meaſured quantity, and draw DB, interſecting CB in B, the other end of the line; then AB meaſured on the ſcale is the diſtance required.

To reſolve it by trigonometry. Subtract the angle ACB from the angle AFB, and you have the angle CAF; then in the triangle CAF, all the angles and the ſide CF are given, therefore to find AF, it will be,

As ſine of the angles CAF : Fig. 64.
to the ſide CF : :
So the ſine of the angle ACF :
to the ſide AF.

Again, ſubtract the angle ADB from AFB, and there remains the angle FBD. Therefore to find FB in the triangle DFB; it is,

As ſine of the angle DBF :
to the ſide FD : :
So the ſine of the angle FDB :
to the ſide FB.

Then in the triangle AFB, there are given the two ſides AF, FB, and the included angle AFB; by which the ſide AB will be found.

If the points C, D, be taken in the lines BF, AF produced, ſo that the angles ACB and ADB may be each of them half the angle AFB; then the line AB will be equal to CD. For then the angle ACF is equal to CAF, and FDB equal to FBD; therefore CF is equal to AF, and DF equal to BF; whence AB is equal to CD.

If there is not room to go back to C and D, you may take a ſtation ſomewhere about G; and find the length of AF (by Prob. VIII.) by the method of meaſuring diſtances, acceſſible only at one end. And the ſame way, find the length of BF. And then the line AB will be found as before.

Otherwiſe thus.

Having choſen two points of ſtation C and D at a proper diſtance, in the ſame plane with AB, whoſe diſtance may be meaſured; and from which, A and B may be ſeen. There ſet up two marks; and placing the inſtrument at C, take the angles ACB, BCD. Then meaſure the ſtationary diſtance CD; and at D, placing the inſtrument, take the angles CDA, ADB. Then you have the angles ACD, and CDB. Take the ſum of the angles ACD and CDA from 180, and there remains the

Fig. 64. the angle CAD. And in the ſame manner the angle CBD is found.

Now we have in the triangle ACD all the angles, and the ſide CD, from which the ſide AC will be found. Then in the triangle CBD, we have all the angles and the ſide CD, whence we can find the ſide CB. Laſtly in the triangle ACB, we have the two ſides AC, BC, and the included angle ACB; from whence the required ſide AB is found.

It may alſo be laid down by rule and compaſs, to ſave the trouble of ſo many calculations. Draw any line as CD, on which ſet off the diſtance of the ſtations at C and D. Then at C, make the angles BCD, BCA, drawing the lines CB, CA. Again make at D, the angles CDA and ADB, all equal to theſe meaſured; and draw DA, which interſects CA in A. And draw DB, which interſects CB in B, and AB drawn and meaſured is the diſtance ſought. And thus you may meaſure any of the lines CA, CB, or DA, DB, by applying them to your ſcale.

It is no matter whether the line AB be horizontal, perpendicular, or inclined, provided CD be in the ſame plane with AB. For then the length of AB may always be determined by the two ſtations C and D. And therefore if AB be an inacceſſible altitude, and C, D, two ſtations either in a perpendicular CD, or upon any hill ſide; the height AB may be found as above; as will be evident by ſuppoſing the plane CADB perpendicular to the horizon.

Otherwiſe thus.

If we cannot get two ſtations C, D, in the ſame plane with AB, the line to be meaſured; then we muſt take at C, the three angles ACB, BCD, and ACD; and at D the three angles CDA, ADB, and CDB; and meaſure CD; and then proceed in the very ſame manner, to find AB, as before. And this may be done two ways, 1. finding AC and CB, and then AB; or 2. finding AD and DB, and then AB. And if it be done both ways it will be a proof of the work. And this method is univerſal, however the ſtations C, D, be taken; and whatever poſition the line AB is in, whether parallel to the horizon, or perpendicular, or any way inclined; provided that A and B can but be ſeen from both the ſtations C and D. Fig. 64.

PROB. X.

To take an inacceſſible diſtance AB, *when both ends of it*, A *and* B, *cannot be ſeen from any two ſtations.*

Aſſume ſome ſtation D, from which both A and B are viſible. Take two other ſide ſtations, C and E, ſo that A may be ſeen from C, and B from E; and both C and E from D. Then by Prob. VIII. find the inacceſſible diſtance DB from the two ſtations D, E; and the inacceſſible diſtance DA, from the ſtations D and C; which is to be done thus. Meaſure the diſtance DE, and the angles BDE, BED, from which find the ſide BD. Alſo meaſure CD, and the angles ADC and ACD; from which the ſide AD will be found. Then meaſure the angle ADB; and in the triangle ADB, there are given the two ſides AD, BD, and the included angle ADB, to find AB as required. 65.

The conſtruction of this, by rule and compaſs, will be eaſy. From any point D, draw DE of a proper length, and at D make the angles EDB, BDA, and ADC; on DC ſet its meaſured diſtance to C. Then at E, make the angle DEB; and at C, the angle DCA. Then the lines EB and CA will cut DB and DA in B and A; therefore AB drawn and meaſured, will be the diſtance ſought.

Fig. 66.

Or thus.

When no ſtation can be found, from which both A and B can be ſeen; take two ſtations C and D, ſo that A may be ſeen from C, and B from D. Then take two ſide ſtations E and F; by help of which, find the inacceſſible diſtances DB and CA (by Prob. VIII.) by meaſuring the diſtance DE, and angles EDB and DEB; and alſo CF and the angles FCA, CFA. Then meaſure the angles ACD, and CDB, and the ſtationary diſtance CD. Then in the triangle ACD, the ſides AC, CD, and included angle ACD being given; the ſide AD and the angle ADC will be found; and ADC ſubtracted from BDC, gives ADB. Then in the triangle ADB, the two ſides AD, DB, and the included angle ADB are given; to find AB the diſtance required.

The conſtruction is eaſy enough. Draw CD, and make the angles FCA, ACD, CDB and BDE; and ſet off the diſtances CF, DE; then make the angles CFA, DEB; and FA, EB, will interſect CA, DB, in A and B; and AB is the diſtance ſought.

Other Problems concerning heights and diſtances might be deviſed, from the various ſituations that objects may be ſuppoſed to have. But the foundation of all is contained in what has been here laid down; and to add more, would only confound the memory, and ſwell the book to little purpoſe.

PROB. XI.

To find the level of two places; or to find the aſcent or deſcent from one place to the other.

This is beſt done by a ſpirit level with teleſcopic ſights; which may be ſet level by ſcrews, which raiſe or lower one end of it. You muſt alſo have two poles, four yards long, with white papers or marks upon them to ſlide up and down, and croſs the middle of the mark, a black line is drawn; and you muſt have two aſſiſtants to carry theſe two poles. Fig. 66.

To take a level from A to I. Let one of the men ſtand at A and the other at C, and at a convenient place as P, between A and C, place the level, and ſet it horizontal by help of the ſcrews. Let the man at A hold the pole upright in his hand at A, whilſt from P, you look through the ſights towards A, and cauſe the man at A to ſlip the mark up or down, till the black ſtroke on the mark at B, be level with the ſights; then meaſure BA. Then cauſe the other man to ſtand at C, and hold the pole upright, then turning the inſtrument round at P, and looking towards C, cauſe him to raiſe or lower his mark, till thro' the ſights, you can ſee the black ſtroke at D; then meaſure the height above the ground CD. And the difference of the heights AB and CD, is the aſcent or deſcent, if there be no more ſtations. But if there are many ſtations make a table with two columns, one for the back ſtations, and the other for the fore ſtations. Now to proceed, ſet down the two heights AB, CD; and let the man at A go to F with his pole, and remove the inſtrument to Q, and level it; then direct the ſights towards C, and let the man at C ſlip his mark up or down till the black line appears thro' the ſights, as at E; and meaſure CE. Then turning the inſtrument about, look towards F, till thro' the ſights you ſee the black mark at G, and meaſure GF; in the mean time the man at C removes to I, and placing the inſtrument at R, direct towards F backward, and to K forwards. And ſo proceed from ſtation to ſtation to the end. And ſet down the meaſures you took of the back ſtations, 67.

Fig. 67. ſtations, AB, CE, F*o*, and the fore ſtations CD, FG, IK, in their proper columns. And at laſt ſum up both the columns, and take the difference of them; and if the fore ſtations exceed, you have a deſcent; but if the back ſtations exceed, it is an aſcent. If they be equal, you have a level.

Sta.	Back ſta.		Fore ſta.	
	f.	in.	f.	in.
1	4	0	2	6
2	5	4	7	3
3	0	0	3	0
	9	4	12	9
			3	5

So as the fore-ſtations are greater, there is a deſcent of 3 feet 5 inches, from A to I. But in the length of a mile, 8 inches deſcent muſt be allowed, upon account of the earth's curvature; that is, if you meaſure along the tangent, at the point A. But if the water is to run, a greater allowance muſt be made, in proportion to the velocity it is to run with. And thus it will be known whether or no, water can be conveyed from one place to another. But it is difficult to take a height, the length of a mile to a few inches. For many propoſitions may be firm and true in geometrical rigor; which reduced to practice amount to nothing at all.

Otherwiſe thus.

To know if water can be conveyed in an open channel; take two boards half a yard long; make two holes in the middle of them, in which ſet up two ſmall ſticks, perpendicular, and exactly of the ſame height; at the tops of which, fix white marks, or pieces of paper. Set the boards in the water to float, at a competent diſtance from one another, the further the better. Then as far from the pond as you can, ſet up a third ſtick, with a mark to ſlip up or down, which muſt be in a line with the other two. Then looking along the tops of the two firſt ſticks, ſlip the mark on the third, up or down, till it be exactly in the ſame horizontal line. Then meaſuring the diſtance of the mark from the ground, and comparing it with the length of the ſticks, you will know, which place is higher, and how much. And thus you may proceed to further places, by ſetting up more ſticks, or removing the boards thither. Fig. 67.

After this manner ſetting your boards in a ditch, and a ſtick at a good diſtance, with a mark on it, in a line with the tops of the other two; you will find if the ground deſcend.

If water is to be conveyed to any aſſigned place, in an open channel, no part of the channel muſt be lower than the place it leads to; for if it is, the water can never riſe again to the height of the laſt place. Therefore, that you may always cauſe it to go by an eaſy deſcent, carry the trench by any turnings and windings, by the ſides of hills or otherwiſe, keeping near a level. Not being ſolicitous to have it go ſtreight, tho' the nearer to a ſtreight line the better, if the ground will permit. But if there happen to be any valleys or low grounds; the water muſt be carried over them in troughs, ſupported by bridges or pillars. And if a hill interferes, it muſt be cut through to the ſame level.

But if water be to be conveyed in cloſe pipes, you may deſcend into valleys, becauſe water in the pipes will riſe to the ſame height. But where there is a hill it muſt be cut thro'. If the deſcent be great, the pipes muſt be laid one up, and another down; or in and out, to check the violent motion of the current; and not in a right line.

Fig. 67. If a canal is to be cut thro' a country, for the benefit of carriage; be ſure to take it from a place high enough to carry the water over the intermediate places; and where there is plenty of water. For you may deſcend when you will, but you cannot riſe; by this means, by turnings and windings, you may conduct it over any riſing grounds. Yet it will often be better to build bridges over hollow places, to carry the canal, than go too far about. And when there is plenty of water, the navigation will be quick; and the water, loſt by opening the locks or flood gates, will ſoon be ſupplied. But if water be ſcarce, theſe defects and loſſes cannot be ſupplied readily enough, which will retard the navigation, and prolong the time; and therefore ſuch a canal can never anſwer the purpoſe.

And tho' we muſt neceſſarily take the water from a height at leaſt equal to the place, where the goods muſt be taken up or laid down; yet it will be a fault to take it higher; becauſe then we ſhall have a greater deſcent to make; the conſequence of which is, that we muſt have more flood gates. And the number of flood gates is known by dividing the whole deſcent by the deſcent of one. And the deſcent at one flood gate may be 4, 5, or 6 feet, according to the bigneſs of the veſſels, and the largeneſs of the canal

68. To illuſtrate the nature of this; let FA be the part of a canal, EC, DB, two flood gates, which may be opened and ſhut at pleaſure; let FED be the ſurface of the water, when the gate DB is ſhut, and EC open. And CBA the ſurface of the water, when EC is ſhut, and DB open. Now a veſſel coming up the part AB, has to make an aſcent into the part EF. In this caſe DB muſt be open, and conſequently the veſſel comes into the part BC, upon the ſame level; and when ſhe is got in there, the gate DB is ſhut, and EC opened, and then the part ECDB inſtantly fills from the part EF; and the veſſel riſes with

the water from the level CB to the level ED, and the Fig. gate EC being open, ſhe can go along the level ſur- 68. face of the water DEF, and thus ſhe has aſcended from the height of AB to the height of EF. And then ſhe can go along EF till ſhe meet with another flood gate, to make another riſe.

In like manner a veſſel coming from F towards E, if EC be open, ſhe goes into the part ED. Then ſhutting EC and opening DB, ſhe goes out of EB, into the part BA, and ſo away.

It may be obſerved, that both gates muſt never be open at once; for then the water in FE would flow into BA, and would all be loſt. And when the gate DB is ſhut, and EC opened, the part EB is filled inſtantly, without ſenſibly decreaſing the height of EF. For EF is a long part of the canal, and EB a very ſhort part, and ſoon filled. And as the loſs of water, is the quantity ECDB; therefore the gates E and D ought to be ſet as near together as they can, to take in one or more veſſels.

I ſhall only add this; when a canal is to be cut thro' a country, it is often attended with difficulties that make it impracticable. For in digging the canal, they often meet with rocks, thro' which they can hardly make any progreſs; and when that is done, the clefts of the rocks will probably take away all the water. Likewiſe when they meet with looſe ouzy ground; by its porous nature it drinks up, and deſtroys a great part of the water. And even when they meet with ſolid ground, yet the expence of digging, levelling, &c. is ſo great, and requires ſo much time to execute it; eſpecially when they are to go many miles in length; that conſidering all things, hardly any of them can afford a profit anſwerable to ſuch a monſtrous expence. When the ground is very much upon the deſcent, a great number of locks or flood gates are required, and therefore a great number of hands will be neceſſary, which

Fig. 68. which loſes a deal of time; and ſuch accidents make the thing extremely difficult, if not impoſſible.

And now having gone thro' all the principal branches of the Mathematics; I ſhall conclude with a few directions, to ſuch as deſign to make theſe Sciences their chief ſtudy; which are by far the moſt difficult, as well as the moſt uſeful parts, of human learning. And firſt, the young ſtudent muſt take care to be well inſtructed in Arithmetick; for this is the baſis of all manner of computations; not only in theſe ſciences, but in all ſorts of buſineſs. In the next place, he muſt get a complete knowledge of Geometry; for on this depends all computations relating to lines, which are infinite. Without a thorough knowledge of this, he will be obliged continually to turn back, or elſe ſtick faſt in the road. Theſe two branches lay the foundation of every thing that is to follow. Then the practical parts may take place, and Trigonometry and the doctrine of the ſphere may be read, which make way for Aſtronomy. And it will be proper then, to ſtudy the Conic Sections, and the higher Geometry, which conſiders the nature of all ſorts of curves. As Algebra is a univerſal ſcience, and applicable to any thing; he may begin to read that, after he is acquainted with Arithmetic and Geometry; but then he can apply it to nothing but numerical and geometrical problems; till ſuch time as he has learned the principles of any other ſcience, that he would apply it to. And in this book of Algebra, there is a ſufficient fund for exerciſing all the branches of the Mathematics. Fluxions being a ſcience of the higheſt nature, and greateſt extent, cannot be attained without the knowledge of all the reſt; and particularly Algebra, by which all computations therein are performed. And this is the method that the young mathematician muſt proceed in, to make the beſt progreſs, and in the Fig. leaſt time. But he muſt frequently expect to meet with obſtacles and difficulties, thro' which his own natural ſagacity muſt carry him, joined with the delight he finds in the knowledge of theſe ſciences. For I doubt he will meet with no other reward for his induſtry, nor even for any diſcoveries he makes, but the pleaſing ſatisfaction of finding out truth. And it is this pleaſure that muſt be his main ſupport under the great fatigue of ſtudying theſe difficult ſciences. For when truth at laſt emerges out of a *chaos* of intricate calculations, and appears in its native brightneſs and luſtre, it poſſeſſes the whole mind; and the labour and fatigue of ſeeking after it inſtantly vaniſhes, and is no more thought on or remembered.

The little encouragement there is in this nation, for promoting theſe ſciences, is the reaſon, that few people go any great length this way. If they have gained ſo much knowledge, as to be able to teach a common ſchool, to get a living by, they think it ſufficient. Few make any farther progreſs; and if they do, they get the pleaſure and pains of it for their labour. Or when a man is eminent for his diſcoveries, perhaps he is dignified with the title of F. R. S. but then he is to pay a quarterly Ceſs for the honour of it. This is the way that ingenuity is rewarded in England.

As to this courſe of Mathematics, that I have here drawn up, it has been the work of my life; for in almoſt all the books I peruſed, I always found either a defect of matter or a defect of method; and ſometimes a redundance of things unneceſſary, and of little conſequence. This put me upon writing, firſt one branch, and then another, till I had got together a tolerable ſyſtem of mathematical learning; where I believe every branch is treated with as much perſpicuity, and contains as much matter, as any one can wiſh for, in that compaſs. For the whole is only deſigned as an introduction for young ſtudents, and to enable them to go thro' larger Treatiſes. Theſe things were all originally deſigned only for my own uſe, and have always proved extremely uſeful to me; and would have been much more ſo, if I had had them ſooner in life. And becauſe what has been ſerviceable to myſelf, I ſuppoſe will be of the ſame ſervice to others, under like circumſtances; therefore I determined to publiſh them. But before this could be done, I had them all to reviſe, correct, and methodize, and to add ſuch things as later improvements had found out; which has coſt me a great deal of time and labour. But then the reader is furniſhed with ſubjects, both eaſy and common, as well as difficult and ſublime; which I expect will fully anſwer his greateſt expectations. However, I have uſed my beſt endeavours. But, if not, I have taken a great deal of pains to no purpoſe; being ſenſible that without pains and patience nothing is to be done: *Sed tandem poſt ardua quies.*

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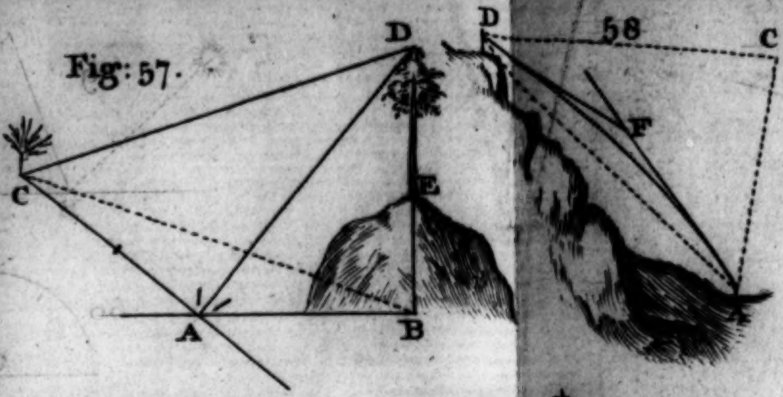
APPENDIX.

Fig: 57. 58



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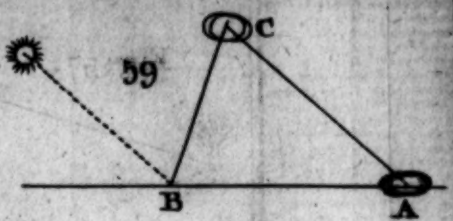

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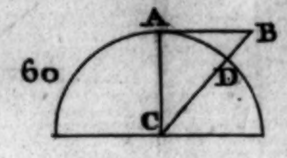

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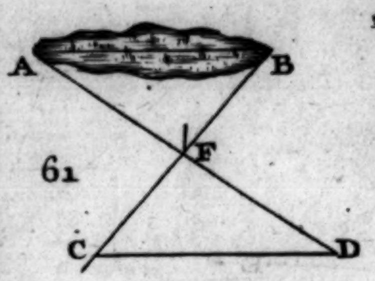

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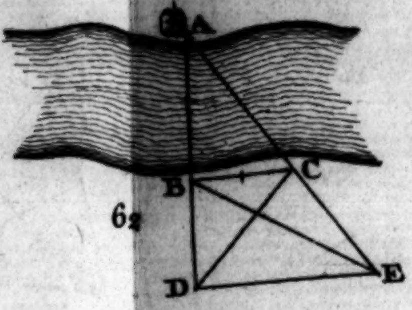

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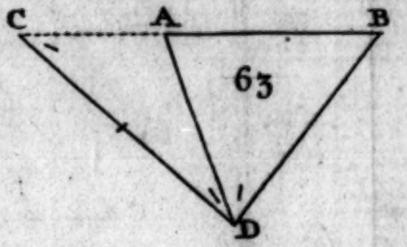

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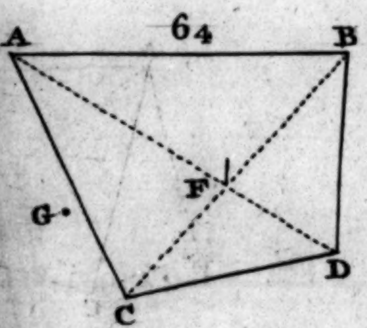

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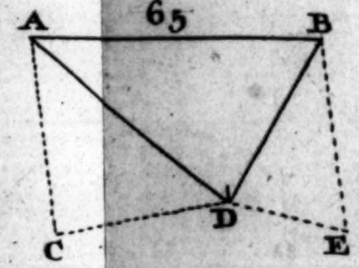

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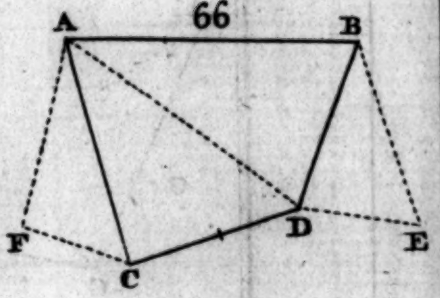

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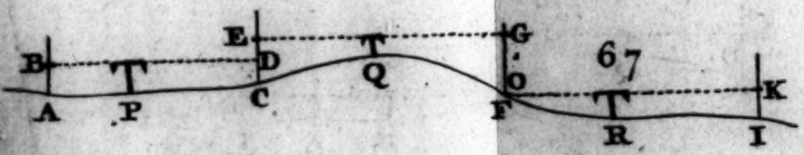

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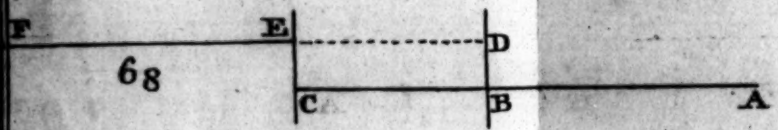


Surveying.

Pl:V. the end

BRITISH MUSEUM

APPENDIX.

Containing Corrections and Additions to all the foregoing Volumes.

ARITHMETIC.

PAGE 7, line laſt; (*read*) of them, taken all together.

Page 31, line 3, &c. The operation being wrong, correct it thus,

```
   C.  ſt.  lb.  C.  ſt.  lb.
6)72   6   11(12   1   1  the quotient.
  72
  --
   0   6) 6(1
          6
         --
          0    6) 11(1
                   6
                  --
                   5 remains.
```

Page 79, l. 8 (*read*) *Ex.* 5.
ib. l. 17 (*read*) *Ex.* 6.
ib. l. 27 (*read*) In *Ex.* 5.
Page 80, l. 18 (*read*) In the 6th *Ex.*
ib. l. 22 (*read*) *Ex.* 7.
Pag. 104, l. 16 (*read*) *the ſingle rule of three.*

Pag. 114, l. 12 b (*read*) of 4 degrees of hardneſs, and 232$\frac{1}{2}$

ib. l. 10 b (*read*) dig a trench, of 7 degrees
ib. l. 6 b (*read*) * 4 deg. —— 7 deg.

Pa. 146, l. 14 b (*read*) to the leaſt error; to be added, if the number was too little; but ſubtracted, if too great. Put out all the 2d article to the bottom of the page, as being needleſs.

Pa. 150, l. 1 (*read*) the correction ... 5.8694077.

ib. after l. 16, *add*,

Note, this rule will anſwer all queſtions that the ſingle rule of falſe will anſwer.

Page 156, l. 10 (*read*) 6 ..... 0.7781512

Pa. 157, l. 8 (*read*) ſquare of 426?

Pa. 170, l. 3 b (*read*) and 38 l. a year

Pa. 171, l. 3 (*read*) 36 ... 684

ib. l. 4, &c. correct it thus,

172)1774(10.32 months
172

54
51

3

P. 186, l. 12 (*read*) two the leaſt whole numbers

ib. l. 13, &c. (*read*) 5 and 3 every where for 365 and 219.

ib. l. 16, 17 (*read*) Therefore the time of meeting will be $3 \times 24\frac{1}{3} = 73$ days.

Or thus,

C = 10
B = 18

diff. 2

Then $2^{m} : 1^{d} :: 73^{m} : 36\frac{1}{2}$ days, the time in which C will overtake B again; then find the leaſt number (by Prob. III. Ch IV. B. II.) which $24\frac{1}{3}$, $14\frac{3}{5}$ and $36\frac{1}{2}$, meaſure; and that is the anſwer, viz. 73.

Pa. 223, l. 8 b (*read*), circulate or come about again, according

Pa. 229, at the end of Prob. I. *add*,

SCHOL.

If one or both numbers (in any operation) be fractional, reduce them to a common denominator. Then proceed with the numerators as before directed; and at laſt, ſubſcribe the common denominator.

All the examples throughout the book ſhould be in italic.

PROPORTIONS.

There wants Sect. I, II, III, all along in the running title.

GEOMETRY.

Page 26, 27, inſtead of Prop. XXV and XXVI. inſert theſe.

PROP. XXV.

If an angle A *of a triangle be biſected by a right line* AD, *which cuts the baſe; the ſegments of the baſe will be proportional to the adjoining ſides of the triangle*; BD : DC : : AB : AC. 1.

And if the external angle EAC *be biſected by the line* Ad, *cutting the baſe in* d; *then the diſtances (of the interſection) from the angles, are as the adjoining ſides*; Bd : Cd : : BA : CA.

Produce BA, and make AE = AC, and draw the line CE; becauſe AE = AC, the $<$ACE = E (Prop. III.) = $\frac{1}{2}$BAC (Prop. I.) = BAD (hyp.); therefore DA, CE are parallels (cor. 3. Prop. IV); and therefore BA : AE or AC : : BD : DC (Prop. XII.)

Again, make C*e* = CA; then the $<$ A*e*C = *e*AC = EA*d* = FAB; therefore *e*C, AB are parallel; whence *d*B : *d*C : : AB : C*e* or CA.

Cor. 1. *If the ſides be as the ſegments of the baſe; the line* AD, *biſects the angle* A.

For ſince BA : AC or AE : : BD : DC; DA and CE are parallels (cor. 1. Prop. XII); and BAD = $<$E, and DAC = ACE = E (Prop. III). Whence BAD = DAC, and A is biſected by AD.

Fig. Cor. 2. *If a line biſecting the vertical angle of a*
1. *triangle, cuts the baſe*; *it will be*

As the ſum of the ſides, BA + AC :
To their difference, BA — AC : :
So the baſe, BC :
To the difference of the ſegments, BD — DC.

For BA : AC : : BD : DC (Prop. XXV), and BA + AC : BA — AC : : BD + DC (BC) : BD — DC or 2DO (Prop. XIII. Proportion); where O is the middle of the baſe BC.

Cor. 3. *Hence* DB : DC : : *d*B : *d*C.

2. Cor. 4. *If* AB, AC, AD, *be continually proportional, and* AE *equal to* AC; *then* CE *drawn, biſects the angle* BCD.

For the triangles ABC, ACD are ſimilar; whence CB : CD : : AB : AC or AE : : AC or AE : AD : : AE — AB (BE) : AD — AE (ED). Therefore (Cor. 1.) angle BCE = ECD.

PROP. XXVI.

3. *If an angle* A *of a triangle* ABC, *be biſected by a right line* AD, *which cuts the baſe*; *the ſquare of the biſecting line, together with the rectangle of the ſegments, is equal to the rectangle of the ſides*; AD^2 + BDC = BAC.

Produce AD, and make the angle <DBP=DAC. Then the 3 triangles CDA, PDB, and PBA are ſimilar. For CAD = PBD = PAB, and CDA = PDB (2.I), whence <C = P, and ADC = ABP (cor. 1. Prop. II). Therefore CD : DA : : PD : BD (Prop. XIII), whence DA × PD = CD × BD (12 Proportion). Again, CA : DA : : AP or AD + DP : AB (Prop. XIII); therefore CA × AB = AD^2 + DA × DP (12 Proportion) = AD^2 + CD × BD (Ax 3).

Cor. *If the external angle* EAC *be biſected by the line* A*d*, *cutting the baſe in d*; *then the rectangle* B*d*C — *the ſquare of the biſecting line* A*d*, *is equal to the rectangle* BAC.

For produce dA to F, and make $\angle dBF = dAC$; then the triangles CdA, FdB, and FBA are ſimilar. For $CAd = dAE = FAB$; and $CdA = FdB$. Whence $ACd = AFB$, and $AdC = ABF$ (cor. 1. Prop. II.) Therefore, $Cd : dA :: Fd : dB$ (XIII); and $dA \times Fd = Cd \times Bd$ (12 Proportion). Again $CA : dA :: AF$ or $dF - dA : AB$ (13). Therefore $CA \times AB = dA \times dF - dA^2$ (12 Proportion) $= Cd \times Bd - dA^2$. Fig. 3.

PROP. XXVII, &c.

Page 73, l. 20, (*read*) *between the ſum and difference of the two radii*;
P. 107, l. 16, (*read*) DFKHCILG =
P. 121, l. 2, (fig. 191)
ib. l. 25, (*read*) DCH, be 3
P. 125, l. 18, fig. 195.
P. 126, l. 2, fig. 195.
ib. l. 8, fig. 196.
ib. l. 19, fig. 197.
ib. l. 4 b, fig. 198.
P. 127, l. 2, fig. 198.
P. 128, l. 2, fig. 198.
ib. l. 2 b, fig. 199, 200.
P. 129, l. 2, fig. 199, 200.
P. 137, l. 5, fig. 209.
P. 156, l. 2, (*read*) *Practical Geometry*, or *The Conſtruction*, &c.
P. 157, 8, Prob. V and VI, (*read*) *To biſect*, *or divide a given*
ib. After Prob. VII, *add*,

Or thus,

If the angle be given in degrees; from the given point B, draw the line BD. Then upon a line of Chords extend your compaſſes from the beginning to 60 degrees (called the *Sweep of* 60), then ſetting one foot in B, with the other deſcribe the arch DC. Then extend from the beginning, to the number

Fig. of degrees given, and ſet that extent from D to C, upon the arch DC. Thro' C draw the line BC, and DBC is the angle required.

Page 162, at the end of Prob. XII. *add,*

Cor. *If it be required to ſet any number of equal parts upon a right line, it is done thus.*

Take with your compaſſes, ſo many equal parts from a diagonal ſcale, or any convenient ſcale of equal parts; and ſet that extent upon your line.

TRIGONOMETRY.

Page 100, after Prop. III. *add,*

Cor. 1. *If two right angled triangles, have the ſame or equal hypothenuſes; the perpendiculars, are as the ſines of the angles at the baſe; and the baſes are as the ſines of the angles at the vertex.*

For they are both in the given ratio of the hypothenuſe (AC) to the radius.

Cor 2. *If two right angled triangles have equal baſes, the perpendiculars are as the tangents of the angles at the baſe; and the hypothenuſes are reciprocally as the ſines of the angles at the vertex.*

For the perpendiculars and tan. oppoſite angles, are in the given ratio of the baſe to the radius.

And if B be the baſe; H, *h*, the hypothenuſes; V, *v*, the ſines of the angles or the vertex. Then by this Prop. as $V : B :: \text{rad} : H = \frac{B}{V} \times \text{rad}$; and $v : B :: \text{rad} : h = \frac{B}{v} \times \text{rad}$. therefore $H : h :: \frac{B}{V} \times \text{rad} : \frac{B}{v} \times \text{rad} :: \frac{1}{V} : \frac{1}{v} :: v : V$.

Cor. 3. *If two right angled triangles have equal perpendiculars; the baſes are as the tangents of the angles at the vertex; and the hypothenuſes are reciprocally as the ſines of the angles at the baſe.*

This is plain by Cor. 2, making the baſe to become the perpendicular. Fig.

Page104, l. 8 b, (*read*) greater ſide :

Page 154, after Prop. XX. *add*,

Corol. *In a right angled ſpherical triangle, if the hypothenuſe be a quadrant, one of the ſides will be a quadrant, and its oppoſite angle a right angle.* fig. 37.

Let BAC be the triangle, A the right angle, the hyp. BC a quadrant. AB (produced at leaſt) paſſes thro' the pole of AC ; and if the quadrant CB be ſuppoſed to revolve about C, it will cut AB in the pole of AC ; but B is the point where they cut ; therefore B is the pole of AC ; and conſequently BA, BC are both quadrants ; and A being a right angle, C will alſo be a right angle (by Prop. XIV).

Page 158, put out the two Corols. at bottom. 4.

Page 159, line 1, (*read*) Cor. 1.

ib. Inſtead of the two Corols. at bottom, inſert

Cor. 1.

In right angled ſpherical triangles ABC, ADF, *that have equal angles at the baſe ; then,*

1. *The ſines of the hypothenuſes, are as the ſines of the perpendiculars,*
2. *The ſines of the baſes are as the tangents of the perpendiculars,*
3. *The tangents of the hypothenuſes, are as the tangents of the baſes.*
4. *The coſines of the hypothenuſes are as the cotangents of the angles at the vertex.*
5. *The coſines of the baſes, are as the coſines of the vertical angles.*
6. *The coſines of the perpendiculars, are reciprocally as the ſines of the angles at the vertex.*

All theſe are eaſily demonſtrated by the help of the two laſt propoſitions ; by extending the ſides of the triangles to quadrants, after the manner of fig. 41, and finding the proportions, in the complemental triangles EDF, ADC, ECB ; which will come out as follows.

Fig. For rad : S.A : : S.AC : S.CB : : S.AF : S.FD.
4. and rad : tan. A : : S.AB : tan. BC : : S.AD : t. DF.
and rad : coſ. A : : tan. AC : tan. AB : : t. AF : t.AD.
and rad : *t*.A : : coſ. AC : : cot. C : : coſ. AF : cot. F.
and rad : S.A : : coſ. AB : : coſ. C : : coſ. AD : coſ. F.
and rad × coſ. A = coſ. CB × S.C = coſ. FD × S.F.

Cor. 2.

In right angled ſpherical triangles, AFB, AFD, *that have equal hypothenuſes; then,*

1. *The ſines of the baſes are as the ſines of the vertical angles.*

2. *The tangents of the baſes, are as the coſines of the angles at the baſe.*

3. *The tangents of the angles at the baſe, are as the cotangents of the angles at the vertex.*

4. *The coſines of the baſes, are reciprocally, as the coſines of the perpendiculars.*

For rad : S.AF : : S.AFB : S.AB : : S.AFD : S.AD.
and rad : t. AF : : coſ. FAB : t. AB : : coſ. FAD : t. AD.
and rad : coſ. AF : : t. FAB : coſ. AFB : : t. FAD : cot. AFD.
and rad × coſ. AF = coſ. AB × coſ. BF = coſ. AD × coſ. DF.

Cor. 3.

In right angled ſpherical triangles ABC, ABF, *having equal baſes; then,*

1. *The tangents of the perpendiculars, are as the tangents of the angles at the baſe.*

2. *The ſines of the angles at the baſe, are as the coſines of the angles at the vertex.*

3. *The ſines of the perpendiculars, are as the cotangents of the vertical angles.*

4. *The coſines of the perpendiculars, are as the coſines of the hypothenuſes.*

5. *The cotangents of the hypothenuſes, are as the coſines of the angles at the baſe.*

6. *The ſines of the hypothenuſes, are reciprocally as the ſines of the vertical angles.*

For,

$$\text{For, rad : S.AB : : }\begin{cases}\text{tan. CAB : tan. CB.}\\ \text{tan. FAB : tan. FB.}\end{cases}$$ Fig. 4.

$$\text{and rad : coſ. AB : : }\begin{cases}\text{S.CAB : coſ. ACB}\\ \text{S.FAB : Coſ. F.}\end{cases}$$

$$\text{and rad : tan. AB : : }\begin{cases}\text{cotan. ACB : S.CB.}\\ \text{cotan. F : S.FB.}\end{cases}$$

$$\text{and rad : coſ. AB : : }\begin{cases}\text{coſ. CB : coſ. AC.}\\ \text{coſ. FB : coſ. AF.}\end{cases}$$

$$\text{and rad : cotan. AB : : }\begin{cases}\text{coſ. CAB : cotan. AC.}\\ \text{coſ. FAB : cotan. AF.}\end{cases}$$

And $\text{rad}\times\text{S.AB} = \text{S.ACB}\times\text{S.AC}$, or $= \text{S.F}\times\text{S.AF}$.

Note, either ſide may be taken for the baſe.

Page 196, after line 7, *add*,

If the ſolution is neither found in ABF nor EDF; produce the ſides from F, thro' A and B to quadrants as before, to find a 3d triangle; and then the ſolution will be found in this new complemental triangle

P. 208, l. 6 b, (*read*) tan. $\frac{1}{2}$AB.

p. 211, l. 7, (*read*) let fall FI a perp.

p. 213, l. 6 b, (*read*) if $AF + FB < 180$.

p. 239, l. 4 b, (*read*) proportionals between

Tab. natural ſines.

8° 38′——ſine .1501106

24°, at bottom right hand, (*read*) 114 deg.

41° 5′——tan. 84 35

43° 53′——ſine .6931922

Tab. artificial ſines.

15° 60′——ſine 9.4403381

29° 60′——coſine 5306

Tab. Logarithms.

Num. 354

Num. 987

1927——log. 8817

Num. 2804

ALGEBRA.

page	line	read
4	7 b	compound quantity ; and the ſum taken all together.
47	1 b	$= \frac{\sqrt[n]{a}}{\sqrt[n]{x}}$.
119	9 b	$agc - bcf$.
148	4	wants a line underneath.
258	8	$a = \frac{d}{b+a} =$
282	6 b	$\frac{.26106}{10.598} =$
300	1	fig. 4.
328	3 b	$a = \frac{3s}{5} =$
336	8	$bfa - bdc$
342	13	$x^9 - x^4 = bx^6$
347	2	fig. 22.
353	9 b	$R^t - 1 = R^t$.
ib.	8 b	$\left(\frac{R^t}{rR^t}\, a = \right)$
358	6	2 miles, the ſecond 3, the third 4,
	9	$\overline{x-1}+2=$ his laſt
364	4	*of the means, or their ſquares* (c) ;
401	1 b	area, and AC $= y$.
408	1	fig. 54, put A for C, B for A, and C for B.
409	5 b	$+\overline{a-c}^2 = 2aa + 2cc =$
413	6	$\overline{s+b}\times\overline{s-b}=$
	7	$z\times\overline{z-a}\times\overline{z-b}\times\overline{z-c}$.
430	12	$\sqrt{tt\times\overline{c+y}^2-}$
441	1 b	GPV, VPM,
442	7	ſpherical triangle ABC,
465	5 b	(mechan. 56 and cor.)
	3 b	*leave out (ib. cor. 5.)*
466	3	$n\times\overline{\sqrt{2}-1}$.

page	line		Fig.
477	1	(mechan. 27. cor. 2)	
	2	dele fig. 132.	
497	10	laſt term will.	
524	1 b	(mec. 18. cor. 3)	
525	16 b	(mec. 23. cor. 1)	
526	10	(mec. 23. cor. 1)	
	6 b	(mec. 23. cor. 1)	

Poſtſcript.

page	line		Fig.
1	13	met with ſuch	
2	7 b	but Dr. Halley and Fr. Robarts, eſq; have	
	5 b	leave them	

ARITHMETIC of INFINITES, &c.

page	line		Fig.
3	1	$= \frac{1}{4}n \times \overline{na}^{3}$, or	
	3 b	$s = \overline{\overline{n + 1 . a}}\vert^{m}$	
7	13	all the *x*'es	
12	11	$c = - A + 2b + C =$	
17	21,	&c. Prop. V. ſhould be italic	
29	4	and thus you may	
44	0 b	FINIS.	

CONIC SECTIONS.

page	line		Fig.
9	17	$CA^2 : CN^2 :: BPA : PM^2$.	4.
18	6 b	(XIII. Cor. 1) $=$	
20		at the end of Prop. 23. *add,* Cor. 1. *rectangle* $NMV = SMF - CE^2$. Cor. 2. $SC^2 : CE^2 ::$ *rectangle* $SMF - CE^2 : PM^2$. fig. 18. For (Prop. 19.) $CA^2 : CE^2 :: PN^2 : PM^2$, and $CA^2 - CE^2 : CE^2 :: PN^2 - PM^2 : PM^2$; that is, $SC^2 : CE^2 ::$ rect. $NMV : PM^2 ::$ (cor. 1.) $SMF - CE^2 . PM^2$.	
29	6	Cor. 1. after line 7, *add,* Cor. 2. $FM : FT :: \sqrt{FMS} : CD$.	

Cor.

Fig. page	line	read
		Cor. 3. *Draw* MO *perp. to the tangent* MH. *Then* $CI \times MO = CD^2$. For (cor. 2. Prop. 35.) MO : CD : : CK : CA : : (cor. 1. of this) CD : CI.
34	5 b	$OD \times DN$: : $CF^2 : CK^2$.
56	7	*more lines el,*
60	2	(ib. Prop. III.)
131	4	multiplying by 4, $QZ^2 =$
		PLATES.
Plate	iii	*f.* 28. draw MO perp. to MH.
plate	x	pa. 66, (at bottom).
pl xii	fig.	102, draw CD, PM.
plxxi		pa. 148, (at bottom).
		CURVE LINES.
89	7	whence y^2
90		after line 5, *add*, Cor. *In a very ſmall part of any curve of finite curvature, as* AG; *the ſubtenſe of the angle of contact, is as the ſquare of the tangent, cord, or arch; for theſe in their evaneſcent ſtate, are all equal; that is,* $BG \propto AB^2$ or AG^2. fig. 86. *It is alſo obſervable, that the cords, ſines, tangents, verſed ſines,* &c. *of a very ſmall arch of a curve, have the ſame properties, as thoſe of the circular arch.*
94	12 b	line MF, are
	7 b	Since $Mo = ms$,
95		at the bottom, *add*,
		SCHOLIUM.
		It may be obſerved, that the circle of equal curvature with the curve at M, is that which paſſes thro' three points of the curve, ſuppoſed to approach infinitely near one another.

For

page	line	read	Fig.
		For two points do not determine it, because an infinite number of circles may paſs thro' two points. But there is only one that can paſs thro' three. And if a circle paſſes thro' two points that approach infinitely near, and at laſt coincide, it only denotes that this circle touches the curve in that point; which an infinite number of circles may do.	
97	3 b	$\dot{q} = \dot{p} + \ddot{p}$;	
98	14 b	*axis*, $\ddot{y} = 0$,	
99	2	$= y\dot{x} \times \frac{\dot{y}\ddot{y}}{\dot{z}}$	
114	6	M*n* $= \dot{x}$, *mn*	
Pl. 8		fig. 100, wants o.	

OPTICS.

page	line	read	Fig.
85	5	$\frac{Cf \times x}{\frac{rr}{2Cf} - x}$	
99		after Cor. 3, *add*, Cor. 4. *In different mediums, it will be,* AB : BC : : AD : DE.	
150	2	*the focal diſtance. And the ſame for a water Lens,* (by Schol. Prop. II.)	
159	14 b	attends old people,	
169	14	$+ \overline{\frac{q-s}{q+s}}\Big\|^{2} t$,	
170	4 b	in the ſame point P; and a ſtop with a hole in it muſt be placed in the middle between AB and CD, or nearer AB, to cut off ſtraggling rays, that colour the object; and then the teleſcope is fit for uſe. The teleſcope——	
173	15	thro' the lens B,	

Fig. page	line	read
179	12	*power of a refracting teleſcope.*
181	3	*thro' a refracting teleſcope.*
184	2 b	*in different refracting te-*
185	15	*In ſimilar refracting teleſcopes*;
193 194 195	1	fig. 116.
203	1	p. 203.
204	5	*leave out*, (by Prop. XVII. B. III.)
	9 b	to do; its magnifying
220	8 b	Microſcope, which is ſhorter. Where
224	7 b	focus of AB, it
pl.viii		fig. 106, GLI ſhould be ſtreight.
pl.ix.		fig. 107, put Q near the end of P.
pl. x		fig. 116, the two axes at C, D ſhould croſs one another.
pl.xiii		fig. 131, the ſquare LMIK to be prick'd.

PERSPECTIVE.

Fig. page	line	read
63	1	Diſtances of 13, &c.
pl. ii		fig. 16, the whole plane RGCQ ſhould be ſhaded.
		fig. 19, for *t* put *l*.
pl. ix		fig. 55, 1, 1 ſhould be at the angles above.
pl. x.		fig. 57, the prick'd lines 41, 1S, wanting, and ſome others.
pl.xv		fig. 77, produce R*l*, a little beyond *l*.

MECHANICS.

Fig. page	line	read
32		After line 11, *add*,

Cor. 5. *The ſpace deſcribed by a falling body in the time of* 1 *vibration* :
is to half the length of the pendulum : :
as the ſquare of the circumference :
to the ſquare of the diameter.

page	line	read	Fig.
		For the ſpaces being as the ſquares of the times, the ſpace fallen : $\frac{1}{2}$length of the pendulum : : as the ſquare of the time of vibration : to the ſquare of the time of falling thro' $\frac{1}{2}$length of the pendulum : : (by this Prop.) as the ſquare of the circumference ; to the ſquare of the diameter.	
35	10	Cor. 1. Hence,	
145	14	Cone ABC	
146		after line 11 b, *add*,	
		I lately contrived another ſort of barometer, which ſhews the aſcent and deſcent of the mercury at the bottom.	
		ABC is a recurve tube, cloſe at the top, where the bucket C is, and open at the end A. The length of CB is 32 or 33 inches, and of AB 6 or 7. The bucket C ſhould contain about as much as the end AB. And the bucket and end CB muſt be quite filled with mercury, as far as B, a little beyond the turn. The wider the bucket C is, the better. The ſcale ſet to the end AB, muſt be graduated downwards, for the mercury falls in this, when it riſes in the other ſort. This being placed againſt a wall, will ſhew the height of the mercury, as in the common ones. And this way is more commodious, as it ſaves the labour of clambering up, upon chairs, to ſee it, as one muſt do, in the common ſort, to ſee exactly.	7.
		A WATER BAROMETER	

PROJECTION.

page	line	read
5	1	fig. 2.
8	18	fig. 5.

off

Fig. page	line	read
9	3 b	off the ſines.
13	17	fig. 12.
14	2 b	the two laſt lines ſhould be roman and indented.
	21	C*p*A + CA*p*;
15	3	ſhould be indented.
		3, 4, 5 lines roman.
16	5	ECI = CAD
18	4	if *p*, *q*, be
	12	harmonically
20	7	projection C,
	9	equal arches EF
21	15	$\frac{OG + OD}{2}$:
25	9 b	points A, B, G,
26	8	draw EB to the point B, and BF perpendicular to it, cutting CH in F. To &c.
27		After *l* laſt *add*, But if the circle DLE was to lie on the other ſide of DGE, the arch HI muſt be taken on the other ſide of H.
29	6 b	interſection P,
30	2	*g*C*t* required.
	4	circle DEH, or
	11 b	pole *p*, draw
		After l. 8 b, *add*, *For a right circle* ACB. Thro' C (*fig.* 34) draw DE perpendicular to AB, then E is the pole of AB. Set the number of degrees from D to H, and draw EGH, then GC contains the degrees required. Or if GC be given, draw EGH, then HD in degrees, is the meaſure of GC. And in like manner AG contains the degrees in AH, or AH is the meaſure of AG.

be

page	line	read	Fig.
31	7 b	be ſet from G to L, and LPE drawn, then,	
33	17	of this circle,	
36	6 b	TV, and	
	2 b	at S cut	
	9 b	any angle $Qst = Qt$.	
		Prop. V. read little *s* throughout.	
37		at the end of Cor. 2. *add,*	
		Cor. 3. *If* P *be the point where the great circles* PC, PF, *&c. meet,* (*as at the pole*); *and if* CB, *the radius of projection, be perpendicular to* CP, *at* C *the point of contact. Then the dividing center* A *of* PF, *is diſtant from* P, *the length of* PB.	5.
		For ICA is perpendicular to PF, and IA = FC; whence $PA^2 = PI^2 + IA^2 = PI^2 + FC^2 = PI^2 + FI^2 + IC^2 = PC^2 + FI^2 = PC^2 + CB^2 = PB^2$, and PA = PB.	
38	2 b	to *o*S;	
41		after l. 16, *add,*	
		Or thus, ſet CD from I to F, and FC from I to A, and draw AP, &c.	
43	2	CL = radius	
44	20	A to *d*,	
45		after l. 17, *add,*	
		Note, the dividing center of all the lines, PM, PL, PN, &c. is always diſtant from P, the length of PW. Therefore all the dividing centers are in the circumference of a circle, whoſe radius is PW, and center P; ſee below, Cor. 2.	
46	12	or $px = P\lambda$;	
	8 b	Cor. 1.	
		after Prop. 15, Cor. 1, *add,*	

Fig.	page	line	read
6.			Cor. 2. *If P be the point where the great circles*, AP, BP, *&c. meet; then to find the dividing centers of all the circles* AP, BP, *&c.* Thro' C the center of projection, draw PCG. Thro' C draw CW perpendicular to PG, and equal to the radius of projection. Draw PW, and WG perpendicular to it, to cut GP in G. At G and the middle of GP, erect the perpendiculars GA and DS. With the radius PW deſcribe the circle EWO. Then to find the dividing center of any circle AP; with the diſtance LP, and one foot in L, cut the circle OWE in R, the dividing center required. For ſince PWG is a right angle, PG is 90 degrees, and ſo is PA. Therefore the dividing center of AP is in a ſemicircle deſcribed round AP, and it is alſo in the circle OWR (by Cor. 3. Prop. V.), and therefore at the interſection R. But DS biſects all theſe circles; and therefore the center of that ſemicircle is in L.
	47	6 b	of meaſures GH, and equal to the radius of projection. To
	49	2, 3	by reaſon ſeveral
	57	11 b	projected from what is

The plates ſhould be paged at the bottom; pl. I, 10. II, 18. III, 22. IV, 24. V, 30. VI, 38. VII, 48. VIII, 50. IX, 54. X, 56. XI, 56. Pl. VII, fig. 44, wants F, and FC.

CENTRIPETAL FORCES.

	1 Pref.	11	irregularities only
	1	11	drawn or impelled to

moved

page	line	read
13	6	moved from A thro' C
	1 b	$= \pi t \sqrt{\frac{2AC}{p}}$.
16	14	whence A*a* × *b*A +
22		after line 5, *add*, Cor. 3. *If* R *be the radius of curvature in* I, *the centripetal force at* I *will be as* $\frac{d}{p^3 R}$ or $\frac{SI}{R \times SP^3}$. For $R = \frac{SI \times IN}{Pq}$, and $\frac{Pq}{IN} = \frac{SI}{R}$; whence the force $\frac{Pq}{PS^3 \times IN} = \frac{SI}{R \times PS^3}$.
28	8	Then univerſally the force is always as
	10	But *ms* or *rs* is
43	15	Such motions as
59	19	P≏C and P♈C.
65	1	4, R recedes from M,
66	4	Curve RDB.
78	6	its axis 1Bz
		Pl. I. fig. 5. the body *m* ſhould be ſhaded.
		Pl. II. fig. 23. P*n* muſt be perp to F*p*.

ASTRONOMY.

page	line	read
103	12	fig. 90.
	3 b	90th degree of
130	1	fig. 31.
162		after l. 4, *add*, Cor. 6. *The horizontal parallax of one body ſeen from another, is equal to the apparent ſemidiameter of this other ſeen from the firſt.* For the parallax of P is the angle at P made by the lines AP, CP continued

Fig. page	line	read
		to the fixt ſtars, and that is equal to the alternate angle APC, under which the radius AC is ſeen, (by Cor. 2), which will be the horizontal parallax, when PAC is a right angle. (fig. 48.)
168		after line 5, *add*, 1. *To find it by Calculation.*
193	8 b	Dec. 30 20 26, 1742
199	8	(becauſe
	9	= 3.1416)
205	7	the planet's angular
	13	the planet's angular
225	5 b	is to its
		after Cor. 3. *add*, Cor. 4. *The velocity at any place* N : *is to the velocity in a circle at the mean diſtance* SA : : $\sqrt{NF}$: $\sqrt{NS}$. For (by Cor. 5. Prop. 16, Centripetal Forces) the velocity in the ellipſis at A, is equal to the velocity in a circle at the diſtance SA.
233	1	fig. 78.
	1 b	♀'s geocent. place
243	15	as p and P,
	10	⊕ place
266	6	or PT.
267		after Cor. 2. *add*, *Or thus.* Let a = apparent diameter of the central body ſeen from the revolving body, and p = the periodic time. Then (Prop. 19. Centr. Forces) the denſity is $\frac{1}{a^3p^2}$. For a is the parallax of the revolving body (by Cor. 6. Prop. 16. Sect. IV.) And the quantity p is known for all the planets and their ſatellites ; and

page	line	read	Fig.
		and *a* may be eaſily found for them all. Whence the denſities are had, without any regard to the ſun's parallax or diſtance. Cor. 3.	
285	13	moved backward.	
318	17	diſtance $ms = MS - Mo + Sp$, very nearly, when MS is large.	
319		after line 3, *add*, But when MS is ſmall or Mm large, Mo will need ſome correction. Therefore to correct it, take $mo = \sqrt{Mm^2 - Mo^2}$), and find its tangent. Then Tan. MS : tan. mo :: $\frac{1}{2}mo$: correction, which muſt be ſubtracted from Mo, when the angle M is acute; but added when obtuſe. When Ss is conſiderable (which ſeldom happens), find Sp ($\sqrt{Ss^2 - Sp^2}$), and add it to mo, before you make the proportion. When DM is greater than a quadrant, the ſigns of Mo and this correction, will be contrary to what they were. The reaſon of this operation in correcting Mo, is, that in the former computation mo is perpendicular to MD, whereas Dmo ſhould be an iſoſceles triangle; and this correction only ſhews how far we are to go beyond o, or how much Mo is to be ſhortened.	
320	16	point *s* is After the Scholium, *add*, *Otherwiſe thus.* The foregoing method will, in moſt caſes, give the anſwer exceeding near the	

Fig. page | line | read

the truth; but being only an approximation, and gained piece by piece, it will deviate a little in ſome particular caſes, as all other patched up methods will do. And therefore it will be better to uſe the following general and exact method, by help of the natural ſines.

Let $Z =$ coſ. $ZM + ZS$, $Z =$ coſ. $Zm + Zs$.
$C =$ coſ. $ZM - ZS$, $c =$ coſ. $ZM - Zs$.
$B =$ coſ. obſerved baſe MS, $b =$ coſ. true baſe *ms*, required.
$P = c - C$, $Q = z - Z$, $R = C - Z$, $S = C - B$, then
$b = B + P + \frac{Q-P}{R}S$, obſerving the ſines, and that the coſines of arches greater than 90, are negative.

For by Cor. 1. Prop. 42. B. III. Trigonometry, $C - Z : C - B :: 2r :$ verſ. $\angle Z = \frac{C-B}{C-Z} \times 2r$, and $c - z : c - b ::$ $2r : \frac{C-B}{C-Z} \times 2r :: C - Z : C - B$.
Whence $\overline{c-z} \times \overline{C-B} = \overline{C-Z} \times \overline{c-b}$.
and $c - b = \frac{\overline{c-z} \times \overline{C-B}}{C-Z}$,
whence $b = c - \frac{\overline{c-z} \times \overline{C-B}}{C-Z}$, which is one rule. But $b = c + \frac{c-z}{C-Z} \times$ $\overline{B-C} = c + \frac{c-z+C-Z-C+Z}{C-Z}$ $\times \overline{B-C} = c + \frac{C-Z}{C-Z} \times \overline{B-C} +$

c—

page | line | read Fig.

$$\frac{c-z+Z-C}{C-Z}\times\overline{B-C}=c+B-C$$

$$+\frac{z-Z-c+C}{C-Z}\overline{C-B}=B+P+$$

$$\frac{Q-P}{R}S.$$

Examp.

Let $\begin{cases} MS = 2^\circ\ 20' \\ ZM = 80\ 21 \\ ZS = 78\ 46 \end{cases}$

moon's parallax $56'\ 10''$
refraction 4 40

$Mm = 51\ 30$

Ss ſtar's refr. 4 40

$ZM = 80\ 21\ 0$
$ZS = 78\ 46\ 0$
$Zm = 79\ 29\ 30$
$Zs = 78\ 50\ 0$

$Z = 159\ 7\ 0$
$z = 158\ 19\ 30$
$C = 1\ 35\ 0$
$c = 0\ 39\ 30$
$P = c - C = .0003158$
$Q = z - Z = 0050144$
$R = C - Z = 1.9339264$
$S = C - B = .0004473$
$Q - P = .0046986$

Coſ.
— .9343082
— .9292938
.9996182
.9999340
. .
.004698
000447
188
18
3
1.93)20.9(11
· 193
16

$B = .9991709$
$P = 3158$
quotient 11

$.9994878 = \text{coſ. } ms = 1^\circ\ 50'\ 2''$

Almoſt all the work conſiſts in ſetting down the data.

fig.

Fig. page	line	read
321	1	fig. 110.
331	0	p. 331.
333	7 b	from the node:
355	16	if x = number
	18	$27.54x = 29.53a$
	6 b	$= x = 50.39^d$
356	7	$= x = 239$ *rcv.*
357		after l. 9, *add*, Put a, x, e, y for the periods of the change, apſis, node, and year; then any of them may be found as before, from the equations; $29.53a = 27.54x = 27.21e = 365.25y$.
367	8	ſooner at London; or weſt if later. Or it is eaſt, if it happen later in reckoning the hours; but weſt if ſooner. This
		In the Plates.
		Plate I. at bottom, pa. 8. Pl. II. pa. 24. Pl. XI. pa. 260.
		Pl. I. fig. 1, the bodies of the Sun, Saturn, Jupiter wanting.
		Pl. VI. fig. 40, the lines ſhould interſect in B.

GEOGRAPHY.

Fig. page	line	read
4	17	length of days
5	9	*Aſcii.* At
	8 b	frigid zone, theſe
19	1,2,3 b	longitude (for lat.)
20	11	and in both ways,
23	1	whoſe diſtance
40	9 b	pole, A and B
57	8	oblate ſpheroid
72	8 b	into a horrible

page	line	read	Fig.
79	9	eaſt by *Spain*,	
82	6	as the land of	
123	2 b	long night be	
128	14	dif. lat. AR.	
	4 b	velocity A*k*.	
130	13	$ts + sr$, &c.	
132	16	ſer than A*z*;	
137	10	whence $z = 2.30$	
144	10	$HI = BE \times \overline{1 + \frac{1}{2}qss}$.	
162	22	fig. 18.	
169		after Cor. 2, *add*, Cor. 3. *And from hence will be known how much the watch is too faſt or too ſlow; which now may be ſet to the true time.*	

DIALLING.

page	line	read	Fig.
9	15	*tag, his rays.*	
16	17	*removed weſtward or eaſtward.*	
17	10	*carried back thro'*	
18	5	*ſituation, on that meridian.*	
58	11	fig. 20.	
	5 b	put out fig. 20.	
77	1	fig. 43.	
131	1	put out fig. 12.	
151		at bottom, *add*, See Cor. 3. Prop. V. Sect. III. Projection.	
163	14 b	Chambalu Pl. IX. fig. 34, 9 and 3 are wrong placed; they ſhould ſtand where *qb*, *qd* cut AB.	

COMBINATIONS.

page	line	read	Fig.
25	5 b	but there ſhould	

MENSURATION.

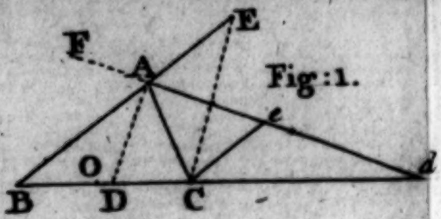
Fig. page	line	read
26	4	37^{l} 12^{s} 6^{d}
66	12	$3 : \frac{9}{2}$
69	1	fig. 8.
89	0 b	dele *Examp.*
118	12	— 1.895091
125	12	9 176.16

SURVEYING.

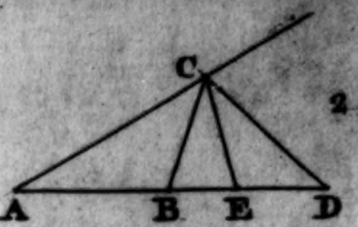
Fig. page	line	read
38	19	angle *abc*, and
89	12 &c.	880 440 110 1.6280

FINIS.

Fig:1.



2



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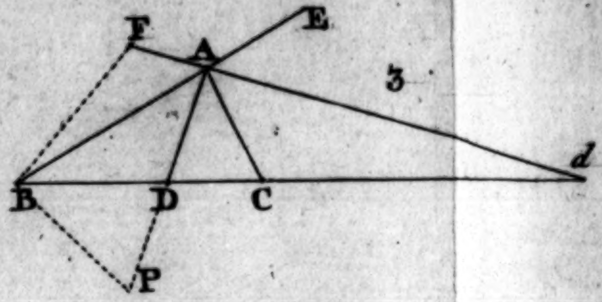

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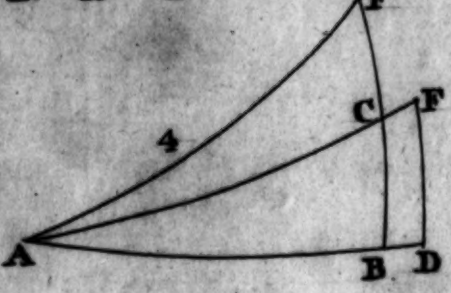

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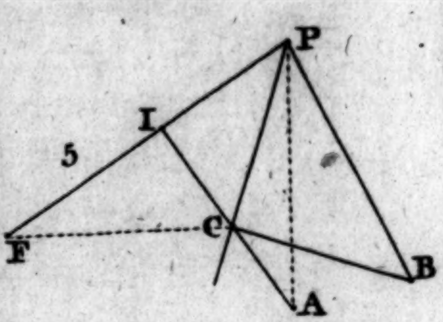

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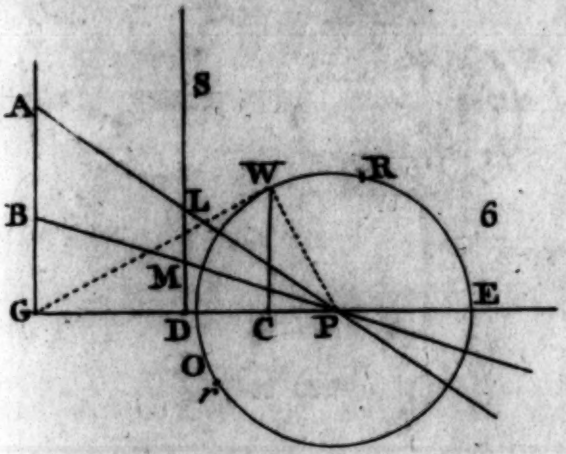
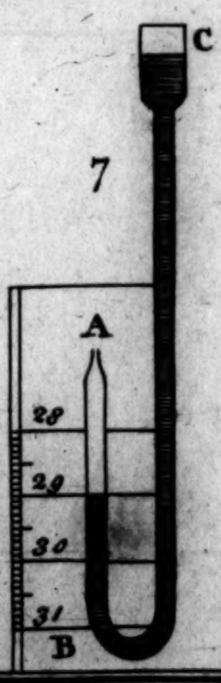


7



Appendix. *the end*